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Abstract

Dong et al. (2007) consider a river network divided into n segments. In each segment there is exactly one agent, who throw some kind of residue into the water. An environmental authority must share the total cost of cleaning the river network among all the agents.

In this paper we propose several rules to distribute the total cleaning-cost among the agents. Moreover, we provide axiomatic characterizations for them using properties based in water taxes. Both, the rules and the characterizations are generalizations of the ones given in Gómez-Rúa (2008).

JEL classification: C71; D61.

Keywords: cost sharing, pollutant-cleaning cost, water taxes.

1 Introduction

Air and water pollution were the initial focus of many environmental policies introduced by OECD countries in the 1970s. These were motivated by a perception that natural environments were being degraded at an accelerating rate, with adverse consequences for ecosystems and human health (OECD, 2008a)

A number of countries (e.g. Australia, France, Spain) aim to manage water resources and pollutant discharges in a common, consistent framework at the river-basin level. An important development in this area is the European Union Water Framework Directive which calls for integrated river basin management planning in all EU member countries by 2009. Because such integrated policies

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clarify the link between water use and water pollution, they are likely to be more efficient in meeting water management objectives. For example, they can enable a comparison between the costs of cleaning water downstream before it is supplied with the costs of discouraging pollution upstream. Integrated policies also facilitate cost recovery (OECD, 2004). When river-basin authorities have access to the cost of treatment for water supply operators, this provides them with a wealth of information on the costs of upstream pollution, which they can use to estimate the rates at which pollutant releases should be charged. River basin management also facilitates water allocation among competing uses within the basin as well as the control of inter-basin transfers. In Spain, river basin authorities are purchasing water rights for over-exploited water bodies (OECD, 2008b).

In Gómez-Rúa (2008) a model is developed in order to study this problem from a theoretical point of view. There is considered a river divide into n segments. There are n agents located along the river whom generate residues. There are proposed several rules to distribute the total pollutant-cleaning cost among all the agents. Furthermore, for each rule is provided an axiomatic characterization using properties based in taxes over water.

This model was introduced by Ni and Wang (2006). They propose two rules to divide the total river-polluting responsibility among the polluters, the Local Responsibility Sharing (LRS) rule, which charges the agent in a given segment his own local costs, and the Upstream Equal Sharing (UES) rule, which charges an agent the sum of the equal divisions of all downstream costs, including his own local costs. The also characterize both rules.

Dong et al. (2007) generalize the results in Ni and Wang (2006) and they consider a river network divided into n segments. In each segment there is exactly one agent, who throw some kind of residue into the water. An environmental authority must share the total cost of cleaning the river network among all the agents. They give many real world examples of sharing a polluted river network. They propose three rules to share this cost. In particular, the Upstream Equal Sharing method is characterized by axioms of *Additivity*, *Independence of Upstream Costs* (that ensures that no agent has any responsibility for the pollution caused in the upstream segments), *Independence of Irrelevant Costs* (that says that an agent shouldn't bear any cost which is irrelevant to her, *i.e.*, that costs with regard to the upstream-downstream relationship in the network), *Efficiency* and *Upstream Symmetry* (which states that for any given downstream costs, all upstream polluters share them equally).

In this paper we propose several rules to distribute the total cleaning-cost among the agents. Furthermore, we provide axiomatic characterizations for them using properties based in water taxes. Both, the rules and the characterizations are generalizations of the ones given in Gómez-Rúa (2008).

We think that Efficiency, Independence of Upstream Costs and Independence of Irrelevant Costs are very appealing properties. However, we introduce a new property in this context, that have into account the idea of the two last properties together. We call this property *Independence of No Responsibility Costs*, and it ensures that an agent's cost share only depends on her own pollution as well as all downstream costs, but not on those costs associated with some other segment for which she has not responsibility.

Additivity has been used in many different situations. For instance, in cooperative games with transferable utility, the Shapley value (Shapley, 1953b), which is considered the most important value in this class of games, is characterized with this property. Moulin (1987) and Chun (1988) used this axiom in surplus problems and in allocation problems, respectively. In bankruptcy problems and other related problems, Bergantiños and Vidal-Puga (2004) characterize up to three different rules with additivity and other properties. This axiom is also used in cost sharing problems (Moulin, 2002). Moulin and Sprumont (2005) focus on additive rules for cost sharing problems with demands. In minimum cost spanning tree problems, Bergantiños and Vidal-Puga (2007) characterize a rule and provided a detailed discussion of this property.

However, as we have discussed in Gómez-Rúa (2008), many situations exist where the Upstream Symmetry cannot be applicable. The main objective of this paper is to change Upstream Symmetry by other suitable properties. Thus, we will obtain rules that can be applicable to these situations.

Our main result is to characterize the set of rules satisfying Efficiency, Additivity and Independence of No Responsibility Costs. Later on, we characterize several rules by adding two properties to the previous one. This two properties were introduced in Gómez-Rúa (2008).

Sometimes, residues that are dumped into the river are biodegradable and therefore pollution disappears over time. Some examples of these are: organic food waste, garden waste, forest residues, farming residues, urban organic waste, etc. In many cases it is possible to know the biodegradation rate of the residues, say δ; so it seems reasonable to demand that the cost that an agent should pay for cleaning a polluted area, depends on this rate. We introduce a new property in this context following this idea called *Biodegradation rate.*

In many countries there are several alternatives in the design of water tax rates. In most cases there exists a difference between the rates applicable to domestic uses and those applicable to industrial uses. A variable component exists which depends on different factors, such as the volume of water consumed, the pollution load, the population of the municipality, type of residue, etc. This is the case of Austria, Canada, Finland, France, Germany, Greece, Hungary, Italy, Korea, Spain, Sweden, USA,... among many others (Gago et al., 2006; OECD, 2006). This idea is collected by the property of *Weighted Tax*.

The last result of the paper is a game theoretic approach. In Gómez-Rúa (2008), we also introduce a TU game, and we proof that one of the rules that we propose coincide with the weighted Shapley value of that game. In this paper, we generalize this result to the new context.

The paper is organizer as follows. In Section 2 we introduce the model. In Section 3 we characterize the family of rules satisfying three properties. In Section 4 we introduce new properties in this context and we present two rules and characterization results for them. Moreover, we prove that one of the rules coincides with the weighted Shapley value of a particular cooperative game.

2 The model

We follow the model presented by Dong et al. (2007).

Consider a river network populated by a set of agents, $N = \{1, 2, ..., n\}$ and a special agent, L, called *lake.* Agents are connected to each other by a serie of links. Upstream agents each exude a certain amount of pollutant to the network. The polluted river needs to be cleaned. The cost of cleaning every river link as well as the lake is known and must be shared among these agents.

Formally, let $N' = N \cup \{L\}$ be the set of agents, and let E be the set of links on N'. We assume that E is a tree, *i.e.*, all agents are connected in E and there is no cycles.

A *cost function* (on the tree E) is a mapping $C : N \cup \{L\} \to \mathbb{R}_+$, where for each $i \in N$, $C(i) = c_i$ is the cost associated with agent i (e.g., the link cost that is associated with the link between agent i and her successor toward L) and $C(L)$ is the cost associated with L. We denote $C(N) = \sum_{i \in N} c_i$.

A *cost-sharing problem on a river network* is a triple (N', E, C).

A *solution* to a problem (N', E, C) is a vector $x = (x_1, ..., x_n, x_L) \in \mathbb{R}_+^{n+1}$ such that $\sum_{i \in N'} x_i = C(N) + C(L)$, where x_i is the cost share assigned to agent $i \in N'$.

A *method* (or *rule*) is a mapping x that assigns to each problem (N', E, C) a solution $x(N', E, C)$.

Given a tree E, the upstream-downstream relation among the agents is uniquely determined by the node L. Also, for any agent, there is a unique path that connects a sequence of downstream agents successively to L.

Now we introduce some notation related with the graph structure. Given (N', E, C), we define the following sets:

$IU(i) := \{j \in N :$ there is a path from j to L such that i is $j's$ immediate downstream agent$\}$. The agents in $IU(i)$ are called *immediate upstream agents* of i in E.

$U(i) := \{j \in N :$ there exists $h_1, h_2, ..., h_m$ in N' such that $h_1 = i$, $h_{k+1} \in IU(h_k)$ for all $1 \leq k \leq m - 1$, and $h_m = j\}$. The agents in $U(i)$ are called *upstream agents* of i in E.

$D(i) := \{j \in N' \colon\ i \in U(j)\}$. The agents in $D(i)$ are called *downstream agents* of i in E.

Given $i, j \in N'$, we define the set $d(i, j) := \{l \in N'$ such that l is in the unique path from i to $j\}$. The *geodesic distance* from i to j is the cardinality of $d(i, j)$, *i.e.*, $|d(i, j)|$.

3 Characterization

Dong, Ni and Wang (2007) characterize a rule, the Upstream Equal Sharing rule with five axioms: Additivity, Efficiency, Independence of Upstream Costs, Independence of Irrelevant Costs and Upstream Symmetry. The last one ensures that all the upstream agents have equal responsibilities for a given downstream pollution cost. However, there exist situations where this axiom cannot be applicable. In Gómez-Rúa (2008) we provide a discussion about this fact.

The property of Independence of Irrelevant Costs ensures that for agent i's cost, any other agent who does not belong to his downstream or upstream area, and does not exude any pollutant should not bear any cleanup cost.

In this section we characterize the set of rules satisfying three properties: Efficiency, Additivity and a new one that capture the ideas of Independence of Upstream Costs and Independence of Irrelevant Costs together. We call this property Independence of No Responsibility Costs.

Before presenting the main result, we introduce formally the axioms.

Efficiency (Eff) $\sum_{i \in N'} x_i = \sum_{i \in N'} c_i$.

Efficiency requires that the cost shares of the agents add up to the total cost.

Additivity (Add) For any $C^1 = (c_1^1, ..., c_n^1, c_L^1) \in \mathbb{R}_+^{n+1}$, $C^2 = (c_1^2, ..., c_n^2, c_L^2) \in \mathbb{R}_+^{n+1}$ and $i \in N'$, $x_i(C^1 + C^2) = x_i(C^1) + x_i(C^2)$.

Independence of No Responsability Costs (INRC) Let $i \in N'$ and $C, C' \in \mathbb{R}_+^{n+1}$ such that $c_j = c'_j$ for all $j \in D(i) \cup \{i\}$. Then, $x_i(C) = x_i(C')$.

This property ensures that an agent's cost share only depends on her own pollution as well as all downstream costs, but not on those costs associated with some other segment. The pollution caused by agent i can not reach these segments, so agent i should not bear any cleanup cost of cleaning these segments.

Now we present the family of rules satisfying *Add*, *Eff* and *INRC*. These rules divide the cost of each segment j (c_j) among the agents responsible for it ($i \in U(j) \cup \{j\}$) proportionally to a vector $p^j \in \mathbb{R}_+^{n+1}$. Namely,

Theorem 1 *A rule x satisfies Eff, Add and INRC if and only if for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $\sum_{i \in N'} p_i^j = 1$ and*

$$x_i(C) = \sum_{j \in N'} p_i^j c_j$$

for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$.

Proof. *We first prove that x satisfies the three axioms:*

x satisfies Eff:

$$\sum_{i \in N'} x_i(C) = \sum_{i \in N'} \sum_{j \in N'} p_i^j c_j = \left(\sum_{j \in N'} c_j\right) \left(\sum_{i \in N'} p_i^j\right) = \sum_{j \in N'} c_j . \blacksquare$$

x satisfies Add: *Let C and $C' \in \mathbb{R}_{+}^{n+1}$ and $i \in N'$. Thus,*

$$\begin{aligned} x_i(C+C') &= \sum_{j\in N'} x_i^j(C+C') \\ &= \sum_{j\in N'} p_i^j\left(c_j + c_j'\right) \\ &= \sum_{j\in N'} p_i^j c_j + \sum_{j\in N'} p_i^j c_j' \\ &= x_i(C) + x_i(C').\blacksquare \end{aligned}$$

x satisfies INRC: *Let $i \in N'$ and $C, C' \in \mathbb{R}_{+}^{n+1}$ such that $c_j = c_j'$ for all $j \in D(i) \cup \{i\}$. Since $p_i^j = 0$ when $i \in (N' \backslash (U(j) \cup \{j\}))$,*

$$\begin{aligned} x_i(C) &= \sum_{j\in N'} p_i^j c_j = \sum_{j\in D(i)\cup\{i\}} p_i^j c_j = \sum_{j\in D(i)\cup\{i\}} p_i^j c_j' \\ &= \sum_{j\in N'} p_i^j c_j' = x_i(C').\blacksquare \end{aligned}$$

We now prove the reciprocal. Assume that x is a solution satisfying Eff, Add and INRC. For each $j \in N'$, let $1_j = (y_1, ..., y_n, y_L) \in \mathbb{R}_{+}^{n+1}$ be such that $y_j = 1$ and $y_i = 0$ when $i \neq j$. We define $p^j = x(1_j)$.

Let x^p be the rule induced by the weight system $\left\{p^j\right\}_{j\in N'}$. We will prove that $x = x^p$ by several claims. The claims are proved following Bergantiños and Vidal-Puga (2004).

Claim 1 *$\left\{p^j\right\}_{j\in N'}$ is a weight system.*

Proof of Claim 1. *Since x satisfies Eff, $\sum_{i\in N'} x_i(1_j) = 1$. By definition of solution, $x_i(1_j) \in \mathbb{R}_{+}^{n+1}$. Let $i, j \in N'$ such that $i \in N' \backslash (U(j) \cup \{j\})$. Since x satisfies INRC, $x_i(1_j) = x_i(0, ..., 0)$. Since $x(0, ..., 0) \in \mathbb{R}_{+}^{n+1}$ and $\sum_{l\in N'} x_l(0, ..., 0) = 0$, $x_i(0, ..., 0) = 0$.* $\blacksquare$

Claim 2 *Let $c_j \in \mathbb{Q}_{+}$ (a non-negative rational number), then $x_i(0, ..., c_j, ..., 0) = c_j x_i(0, ..., 1, ..., 0)$.*

Proof of Claim 2. *Let $c_j = 1/q$, where $q \in \mathbb{N}$. By Add, $x_i(0, ..., 1, ..., 0) = \sum_{k=1}^{q} x_i(0, ..., \frac{1}{q}, ..., 0) = q x_i(0, ..., \frac{1}{q}, ..., 0)$. Thus,*

$$x_i\left(0, ..., \frac{1}{q}, ..., 0\right) = \frac{x_i(0, ..., 1, ..., 0)}{q} = c_j x_i(0, ..., 1, ..., 0). \tag{1}$$

Let $c_j \in \mathbb{Q}_{+}$, say $c_j = \frac{p}{q}$. By Add,

$$x_i\left(0, ..., \frac{p}{q}, ..., 0\right) = p x_i\left(0, ..., \frac{1}{q}, ..., 0\right).$$

Then by (1),

$$x_i\left(0, ..., \frac{p}{q}, ..., 0\right) = \frac{p}{q} x_i(0, ..., 1, ..., 0).\blacksquare$$

Claim 3 *Let* $c_j \in \mathbb{R}_+ \backslash \mathbb{Q}_+$ *(a non-negative irrational number), then* $x_i(0,...,c_j,...,0) = c_j x_i(0,...,1,...,0)$.

Proof of Claim 3. *Let* $c_j \in \mathbb{R}_+ \backslash \mathbb{Q}_+$. *Then, there exists* $\{b_l\}_{l=1}^{\infty}$ *such that* $b_l \in \mathbb{Q}_+$, $b_l < c_j$ *and* $\lim_{l\to\infty} b_l = c_j$.

Let $l \in \mathbb{N}$. *Since* $x(0,...,c_j-b_l,...,0) \in \mathbb{R}_+^{n+1}$ *and* $\sum_{i\in N'} x_i(0,...,c_j-b_l,...,0) = c_j - b_l$,

$$0 \leq x_i(0,...,c_j-b_l,...,0) \leq c_j - b_l.$$

By Add, $x_i(0,...,c_j,...,0) = x_i(0,...,c_j-b_l,...,0) + x_i(0,...,b_l,...,0)$. *So,*

$$0 \leq x_i(0,...,c_j,...,0) - x_i(0,...,b_l,...,0) \leq c_j - b_l.$$

Since $b_l \in \mathbb{Q}_+$, $x_i(0,...,b_l,...,0) = b_l x_i(0,...,1,...,0)$. *Then,*

$$0 \leq x_i(0,...,c_j,...,0) - b_l x_i(0,...,1,...,0) \leq c_j - b_l.$$

Thus,

$$0 \leq \lim_{l\to\infty} \left[x_i(0,...,c_j,...,0) - b_l x_i(0,...,1,...,0)\right] \leq \lim_{l\to\infty} \left[c_j - b_l\right].$$

So,

$$0 \leq x_i(0,...,c_j,...,0) - c_j x_i(0,...,1,...,0) \leq 0.$$

Therefore,

$$x_i(0,...,c_j,...,0) = c_j x_i(0,...,1,...,0).\blacksquare$$

Claim 4 *Given* $i \in N'$ *and* $C \in \mathbb{R}_+^{n+1}$, $x_i(c_1,...,c_n,c_L) = \sum_{j\in N'} x_i(0,...,0,c_j,0,...,0)$.

Proof of Claim 4. *It follows from the fact that x satisfies Add.*$\blacksquare$

Since $x_i^p(c_1,...,c_n,c_L) = \sum_{j\in N'} p_i^j c_j$, *and by Claims 2 and 3,* $x_i(0,...,c_j,...,0) = c_j x_i(0,...,1,...,0) = c_j p_i^j$ *for all* $j \in N'$ *and all* $c_j \in \mathbb{R}_+$, *it is clear that* $x = x^p$.$\blacksquare$

$\blacksquare$

4 Other results

In this section, we provide characterizations of new rules, adding different properties based on possible and real taxes over pollution in Theorem 1. These properties are generalizations of the ones introduced in Gómez-Rúa (2008).

As we have pointed in that paper, in many cases all the agents throw the same kind of residues into the water. Furthermore, the residues are biodegradable and thus the pollution disappears over time; for instance: organic food waste, garden waste, forest residues, some industrial waste... In many occasions it is possible to know the biodegradation rate of the residues, say δ. If it happens, the cost that an agent pays for a polluted area should depend on this biodegradation rate. We introduce a new property following this idea:

Biodegradation Rate (BR) Given $j \in N'$, for any $i,k \in U(j) \cup \{j\}$ such that $|d(i,j)| \geq |d(k,j)|$,

$$x_i(0,...,0,c_j,0,...0) = \delta^{|d(i,j)|-|d(k,j)|} x_k(0,...,0,c_j,0,...0).$$

We assume that $0 \leq \delta \leq 1$. Notice that $\delta = 0$ means that the residue of agent i only affects its own area. In this case BR means that every agent pays the cost corresponding to its own area, namely $x_i(C) = c_i$ for all C and $i \in N'$. Furthermore, $\delta = 1$ means that the residue is non-biodegradable. In this case BR coincides with Upstream Symmetry (Dong et al., (2007)).

In the next theorem we study the effects of adding BR to the properties in Theorem 1.

Theorem 2 . *A rule x satisfies Add, Eff, INRC and BR if and only if for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $p_i^j = \delta^{|d(i,j)|-|d(k,j)|} p_k^j$ for any $i \in U(j)$, $k \in U(j) \cup \{j\}$ such that $|d(i,j)| \geq |d(k,j)|$, $\sum_{i \in N'} p_i^j = 1$ and*

$$x_i(C) = \sum_{j \in N'} p_i^j c_j$$

for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$.

Proof. *We first prove that x satisfies* BR. *Let $i, j, k \in N'$ such that $i \in U(j)$, $k \in (U(j) \cup \{j\})$ and $|d(i,j)| \geq |d(k,j)|$. Let $(0, ..., c_j, ..., 0) \in \mathbb{R}_+^{n+1}$. Then,*

$$\begin{aligned} x_i(0, ..., c_j, ..., 0) &= \sum_{l \in N'} p_i^l c_l = p_i^j c_j = \delta^{|d(i,j)|} p_j^j c_j \\ &= \delta^{|d(k,j)|} \delta^{|d(i,j)|-|d(k,j)|} p_j^j c_j = \delta^{|d(i,j)|-|d(k,j)|} p_k^j c_j \\ &= \delta^{|d(i,j)|-|d(k,j)|} x_k(0, ..., c_j, ..., 0). \blacksquare \end{aligned}$$

We now prove the reciprocal. Let x be a rule satisfying Add, Eff, INRC and BR. By Theorem 1 for each $j = 1, ..., n, L$, there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $\sum\limits_{i \in N'} p_i^j = 1$ and $x_i(C) = \sum\limits_{j \in N'} p_i^j c_j$ for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$. We now prove that $p_i^j = \delta^{|d(i,j)|-|d(k,j)|} p_k^j$ for any $i \in U(j)$, $k \in U(j) \cup \{j\}$ such that $|d(i,j)| \geq |d(k,j)|$.

Let $i, j, k \in N'$ such that $i \in U(j)$, $k \in (U(j) \cup \{j\})$ and $|d(i,j)| \geq |d(k,j)|$. By the proof of Theorem 1, $p^j = x(1_j)$. Since x satisfies BR,

$$\begin{aligned} p_i^j &= x_i(1_j) = \delta^{|d(i,j)|} x_j(1_j) = \delta^{|d(k,j)|} \delta^{|d(i,j)|-|d(k,j)|} x_j(1_j) \\ &= \delta^{|d(i,j)|-|d(k,j)|} x_k(1_j) = \delta^{|d(i,j)|-|d(k,j)|} p_k^j. \blacksquare \end{aligned}$$

■

In many countries, such as like Spain, Austria, Canada, Finland, France, Germany, Greece, Hungary, Italy, Korea, Sweden, USA,... (OECD, 2006), there exists a difference between the rates applicable to domestic uses and those applicable to industrial ones. In particular, most of the autonomous regional governments of Aragon, Catalonia, Madrid, Galicia, Murcia, Navarre and La Rioja in Spain, determine the base of the tax for industrial use by estimating or directly measuring the pollution load (Gago et al., 2005). Further, as previously highlighted, in Valencia and Catalonia the rates applicable to domestic uses consider the population of the municipality. In all these situations the taxes can be modulated considering different factors, such as pollution load, population of the cities, monthly water consumption, etc. (See Gago et al., 2006). In Gómez-Rúa (2008) we introduce a property that captures these ideas. Now, we generalize that property for the context of a network river with the following axiom:

Weighted Tax with respect to w (WT-w) Let $w = (w_i)_{i \in N'} \in \mathbb{R}_+^{n+1}$. We say that x satisfies WT with respect to w if for any $i, j, k \in N'$ such that $i \in U(j)$, $k \in U(j) \cup \{j\}$,

$$\frac{x_i(0, ..., 0, c_j, 0, ..., 0)}{x_k(0, ..., 0, c_j, 0, ..., 0)} = \frac{w_i}{w_k}.$$

This property states that the amount that each agent pays for a polluted area is given by some exogenous factor.

WT generalizes *Upstream Symmetry* because when $w_i = w_j$ for all $i, j \in N'$, both properties coincide.

In the next theorem we study the effects of adding *WT* to the properties in Theorem 1.

Theorem 3 . *A rule x satisfies Add, Eff, INRC and WT if and only if for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $p_i^j = \frac{w_i}{\sum_{l \in U(j) \cup \{j\}} w_l}$ for all $i \in U(j) \cup \{j\}$ and*

$$x_i(C) = \sum_{j \in N'} p_i^j c_j$$

for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$.

Proof. *We first prove that x satisfies* WT*. Let $i, j, k \in N'$ such that $i \in U(j)$, $k \in U(j) \cup \{j\}$. Let $(0, ..., c_j, ..., 0) \in \mathbb{R}_+^{n+1}$. Then,*

$$\begin{aligned} x_i(0, ..., c_j, ..., 0) &= \sum\nolimits_{l \in N'} p_i^l c_l = p_i^j c_j = \frac{w_i}{\sum_{l \in U(j) \cup \{j\}} w_l} c_j \\ &= \frac{w_i}{w_k} \frac{w_k}{\sum_{l \in U(j) \cup \{j\}} w_l} c_j = \frac{w_i}{w_k} p_k^j c_j = \frac{w_i}{w_k} x_k(0, ..., c_j, ..., 0). \blacksquare \end{aligned}$$

We now prove the reciprocal. Let x be a rule satisfying Add, Eff, INRC and WT. By Theorem 1 for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $\sum\limits_{i \in N'} p_i^j = 1$

and $x_i(C) = \sum_{j\in N'} p_i^j c_j$ *for all* $C \in \mathbb{R}_+^{n+1}$ *and all* $i \in N'$. *We now prove that* $p_i^j = \frac{w_i}{\sum_{l\in U(j)\cup\{j\}} w_l}$ *for any* $i \in U(j)$, $k \in U(j) \cup \{j\}$.

Let $i, j, k \in N'$ *such that* $i \in U(j)$, $k \in U(j) \cup \{j\}$. *By the proof of Theorem 1,* $p^j = x(1_j)$. *Since* x *satisfies Eff and WT,*

$$\frac{1}{p_j^j} = \frac{\sum_{l\in U(j)\cup\{j\}} p_l^j}{p_j^j} = \sum_{l\in U(j)\cup\{j\}} \frac{x_l(1_j)}{x_j(1_j)} = \sum_{l\in U(j)\cup\{j\}} \frac{w_l}{w_j} = \frac{\sum_{l\in U(j)\cup\{j\}} w_l}{w_j}.$$

By WT, for each $i \in U(j)$

$$p_i^j = x_i(1_j) = \frac{w_i}{w_j} x_j(1_j) = \frac{w_i}{w_j} p_j^j = \frac{w_i}{\sum_{l\in U(j)\cup\{j\}} w_l}. \blacksquare$$

■

Dong et al. (2007) give a game theoretic approach for the problem and they study the relationships between the rules they propose and the Shapley value (Shapley, 1953). They consider three different games, defined from the river network problem. In Gómez-Rúa (2008), we also introduce a TU game, and we proof that one of the rules that we propose coincide with the weighted Shapley value of that game. Now, we generalize this result to the new context.

We now relate the solutions given by Theorem 3 with the weighted Shapley values of a *TU* game.

Given a problem (N', E, C) we define the *TU* game $\left(N', v^{E,C}\right)$ where

$$v^{E,C}(S) = \sum_{i\in S: U(i)\cup\{i\}\subset S} c_i$$

for all $S \subset N'$. Namely $v^{E,C}(S)$ represents the pollutant-cleaning costs in the segments polluted only by agents in S. This definition implies that, if a segment i is polluted by agents that are in S but also by agents that does not belong to S, then the segment i is not took into account in order to compute $v^{E,C}(S)$.

Theorem 4 . *Let* x^w *the solution given by Theorem 3. Then,* x^w *coincide with the weighted Shapley value of* $v^{E,C}$ *with weights given by* $w \in \mathbb{R}_{++}^N$, $\phi^w\left(N', v^{E,C}\right)$.

Proof. *Let* $w = (w_i)_{i\in N'} \in \mathbb{R}_+^{N'}$. *Let* $\{u_S\}_{S\subset N'}$ *be a family of TU games such that* $u_S(T) = 1$ *if* $S \cap T \neq \varnothing$ *and* $u_S(T) = 0$ *otherwise. It is well known that* $\{u_S\}_{S\subset N'}$ *is a basis for the set of all TU games. Kalai and Samet (1987) define the value* ϕ^{w*} *as the unique linear value satisfying that for each* $S \subset N'$, $\phi_i^{w*}(u_S) = \frac{w_i}{\sum_{k\in S} w_k}$ *if* $i \in S$ *and* $\phi_i^{w*}(u_S) = 0$ *otherwise. Moreover, they prove that for each* $w \in \mathbb{R}_+^{N'}$ *and each TU game* v, $\phi^{w*}(v) = \phi^w(v^*)$ *where* $v^*(S) = v(N') - v(N'\backslash S)$ *for all* $S \subset N'$.

Given (N', E, C), *for each* $j \in N'$, *let* $\left(N', v^j\right)$ *be the TU game where for all* $S \subset N'$, $v^j(S) = c_j$ *if* $S \cap (U(j) \cup \{j\}) \neq \varnothing$ *and* $v^j(S) = 0$ *otherwise. Notice that* $v^j = c_j u_{\{U(j) \cup \{j\}\}}$ *for all* $j \in N'$.

Given $i \in N'$,

$$\begin{aligned} x_i^w(C) &= \sum\nolimits_{j \in N'} p_i^j c_j = \sum\nolimits_{j \in D(i) \cup \{i\}} \frac{w_i}{\sum_{k \in U(j) \cup \{j\}} w_k} c_j \\ &= \sum\nolimits_{j \in D(i) \cup \{i\}} \phi_i^{w*}\left(v^j\right) = \sum\nolimits_{j \in N'} \phi_i^{w*}\left(v^j\right) \\ &= \sum\nolimits_{j \in N'} \phi_i^w\left(v^{j*}\right) = \phi_i^w\left(\sum\nolimits_{j \in N'} v^{j*}\right). \end{aligned}$$

Let $S \subset N'$. *Then,* $v^{j*}(S) = v^j(N') - v^j(N' \backslash S) = c_j - v^j(N' \backslash S)$. *Since* $v^j(N' \backslash S) = c_j$ *when* $(N' \backslash S) \cap (U(j) \cup \{j\}) \neq \varnothing$ *and* $v^j(N' \backslash S) = 0$ *when* $(N' \backslash S) \cap (U(j) \cup \{j\}) = \varnothing$,

$$v^{j*}(S) = \begin{cases} c_j & \text{if } \{U(j) \cup \{j\}\} \subset S \\ 0 & \text{otherwise.} \end{cases}.$$

Now it is trivial to prove that for all $S \subset N'$, $v^{C,E}(S) = \sum_{j \in N'} v^{j*}(S)$. *Hence,* $x_i^w(C) = \phi_i^w\left(v^C\right)$. ■

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