

Financial globalization and economic growth

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Abstract

Capital mobility leads to a smaller speed of convergence in an open economy than in a closed economy, using a two-sector endogenous growth model. This is related to the presence of two capitals, produced with specific technologies, and where one of the capitals is nontradable, like infrastructures or human capital. Suppose, for example, that the economy is relatively less abundant in human capital, leading to a decrease of the remuneration of this capital during the transition. In a closed economy, the remuneration of physical capital will be increasing during the transition. In the open economy, the alternative investment yields the international interest rate, corresponding to the steady state net remuneration of physical capital in the closed economy. The non-arbitrage condition shows a larger difference in the remuneration of the two capitals in the closed economy. It leads to a higher accumulation of human capital and thus to a faster speed of convergence in the closed economy. This result stands in sharp contrast with that of the one-sector neoclassical growth model, where the speed of convergence is smaller in the closed economy.

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1 Introduction

Financial globalization has increased since 1970, with a more pronounced path after the mid-eighties. Developed countries have been opening the capital account since the mid-eighties or the beginning of the nineties. In developing countries, emerging markets have a clear pattern of financial opening since the beginning of the nineties or mid-nineties¹. There is a growing literature on the empirical effects of financial openness on growth. In a survey of the studies on this issue, Kose et al. (2006) conclude that there is not a robust and positive effect of international financial integration on growth, although financial globalization may have an influence on growth through indirect channels.

Another related issue is the effect of financial globalization on the speed of convergence. The studies in the survey referred to above do not look directly at this effect, but some of those studies examine interaction terms of financial openness with specific conditions of the economy. In this line, Edison, Levine, Ricci and Sløk (2002) present growth regressions with five proxies of financial openness and their interaction with the logarithm of initial GDP in a cross-section of countries for the period 1980-2000². The coefficient associated with the interaction term for the different proxies is in general not significant and it is negative or positive. Following this study, there is no robust effect of financial globalization on the speed of convergence.

In this paper we want to analyze the effects of capital mobility in the speed of convergence. This is a much less studied issue than trade openness and the speed of convergence. We compare the speed of convergence of closed and open economies, using a two-sector endogenous growth model with two capitals. On the one hand physical capital, and on the other hand infra-structures or human capital. In the open economy, physical capital is tradable and human capital is nontradable. Although this strategy can suggest only an increase in the complexity of the analysis, what happens

¹These patterns are presented in Kose, Prasad, Rogoff and Wei (2006) using two proxies for financial openness. See Figure 3 in their paper. A de jure proxy measures restrictions on capital account and equals one if there are no restrictions. The data are from the IMF's Annual Report on Exchange Arrangements and Exchange Restrictions. A de facto proxy is based on the ratio of gross stocks of foreign assets and liabilities to GDP of a country. It is based on Lane and Milesi-Ferretti (2006).

²See Table 4, Panel A, in their paper. From these regressions one can derive the effect of financial openness on the speed of convergence. The authors present the results with OLS, but point out that these results do not change with two-stage least squares and panel regressions.

is that one can understand the transitional dynamics using a non-arbitrage condition where the difference in the remuneration of both capitals gives the incentive to accumulate relatively more in one or another capital. With a two-sector two-factor model, the remuneration of both capitals only depends on the relative price. It follows that we have a differential equation of the relative price that only depends on the relative price. This non-arbitrage condition provides the transitional dynamics of the model in a simple way.

Moreover, we arrive at a surprising result. The speed of convergence of the open economy is smaller than that of the closed economy. This result stands in sharp contrast with that of the one-sector neoclassical growth model, where the speed of convergence is smaller in the closed economy.

Suppose a closed economy that initially has relatively more physical capital than human capital³. The remuneration of human capital is relatively higher than that of physical capital and there is an incentive to invest relatively more in human capital. As human capital becomes relatively more available, its relative price (the price of human capital over the price of physical capital) decreases. During the transition to the steady state the remuneration of human capital (whose production is more intensive in human capital) will be decreasing while the remuneration of physical capital will be increasing, until we arrive at the steady state investment rate.

We consider a small open economy with the same structure of production of the closed economy. Now consumption goods and physical capital are tradable and human capital is nontradable. The convergence of the economy can be analyzed again with the non-arbitrage condition, where now the alternative to investing in human capital is a foreign bond yielding the international interest rate. As for the closed economy suppose that at time zero physical capital is relatively more abundant than human capital. During the transition, the remuneration of human capital will be higher than the international interest rate and the latter higher than the remuneration of physical capital. It follows that the non-arbitrage condition gives an incentive to invest relatively more in human capital, but this incentive is higher in the closed economy than in the open economy. The higher incentive in the closed economy to invest relatively more in human capital also leads to a speed of convergence faster in the closed economy than in the open economy.

The intuition presented above was derived with a closed economy without adjustment costs, while these costs are present in the open economy. One could ask if this result is only valid for small values of the adjustment costs.

³We would like to point out that the analysis would be the same if there was relatively more human capital available.

With the aim of comparing exactly the same structure of production, we develop a closed economy with adjustment costs. We analytically derive its speed of convergence, when the share of physical capital in the production of human capital goes in limit to zero. We show that our result applies for any finite value of the adjustment costs, in an economy with incomplete specialization. Using values for the adjustment costs which are considered in the literature, we also show in a numeric experiment that our result is robust to the inclusion of physical capital in the production of human capital.

Our paper is related to several important strands of theory on growth, convergence and openness. First there is the literature on two-sector endogenous growth models, as Lucas (1988) and Rebelo (1991). The characterization of transitional dynamics of these models is seen in Bond, Wang and Yip (1996) and Mulligan and Sala-i-Martin (1993). Ortigueira and Santos (1997) arrive at an analytical expression for the speed of convergence of a two-sector endogenous growth model based on a theorem presented in the appendix of their paper. However, they do not derive an expression for the speed of convergence of a closed economy with adjustment costs. A second strand, with tradition in international economics, makes a distinction between tradable and nontradable goods. For one of the first formalization in the spirit of our model see Bruno (1976). Brock and Turnovsky (1994) characterize the dynamics of this exogenous growth model, with two sectors and three factors of production (labor and two accumulable capitals). But they are not interested in the speed of convergence of this open economy⁴. Farmer and Lahiri (2002) analyze the transitional dynamics of a two countries world economy. These authors use a two-sector endogenous growth model and the distinction tradables-nontradables, but they do not present analytical expressions for the speed of convergence. Turnovsky (1996) develops a dependent economy using a two-sector endogenous growth model, but he is not analyzing the speed of convergence. He is interested in the dynamics of the model following a shock in the parameters. Finally, our paper can be related to the literature on open economies and growth, which concentrates more on trade in goods. See, for example, Grossman and Helpman (1991) for the effects of trade in the incentives to innovate. Papers that also characterize the transitional dynamics are Ventura (1997) and Acemoglu and Ventura (2002). The former does not have capital mobility, based on a form of factor price equalization. In the latter there is no capital mobility by assumption and they establish a positive link between trade openness

⁴Notice that one can not derive analytically the speed of convergence of a closed economy with the same structure of production.

and speed of convergence. A model that assumes technological spillovers from advanced countries to backward countries and derives its transitional dynamics is presented in Howitt (2000). Obstfeld (1994) use a stochastic version of the AK model to illustrate how world capital market integration can raise steady-state growth.

The paper is organized as follows. Section 2 presents a two sector endogenous growth model and its dynamics. Section 3 develops a closed economy version with adjustment costs. In Section 4 we present a small open economy with adjustment costs. In Section 5 we will compare the speed of convergence of the open economy with the speed of a closed economy, both with adjustment costs. Concluding remarks are presented in Section 6. The Appendix contains details of the derivation of some analytical results.

2 A two sector endogenous growth model

This section presents the structure of a two sector endogenous growth model and derives an analytical solution for its speed of convergence. We also provide an intuition of the behaviour of the transitional dynamics, based on a non-arbitrage condition for the accumulation of physical and human capital.

We will use this analytical solution of the speed of convergence and the intuition associated with it when comparing the speed of convergence of the closed and the open economies. Appendix A contains most of the characterization of this model, as it has been studied by Bond et al. (1996).

2.1 The model

The utility function of the representative agent is given by:

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma} \quad (1)$$

where C represents consumption, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution. For $\sigma = 1$, we have $U(C) = \log C$.

There are two sectors of production, one (Y^K) for consumption goods and accumulation of physical capital, \dot{K} , and the other (Y^H) for accumulation of human capital, \dot{H} . The technologies are similar to the ones in Lucas (1988), but here the production of human capital also uses physical capital⁵.

⁵For an analysis of this general model and its dynamics see Bond et al. (1996) and Mulligan and Sala-i-Martin (1993). The latter characterize the transitional dynamics

There are no externalities.

We have

$$\dot{K} = Y^K - \delta_K K - C = A(vK)^\alpha (uH)^{1-\alpha} - \delta_K K - C, \quad (2)$$

$$\dot{H} = Y^H - \delta_H H = B[(1-v)K]^\beta [(1-u)H]^{1-\beta} - \delta_H H. \quad (3)$$

The depreciation rate is δ_i , with $i = K, H$, for physical capital and human capital, respectively. α ($0 < \alpha < 1$) is a parameter with $1 - \alpha$ representing the share of nontradable capital in the consumption sector; v ($0 \leq v \leq 1$) is the fraction of the physical capital and u ($0 \leq u \leq 1$) is the fraction of the human capital used in the consumption sector. A and B represent the level of technology in the consumption and the human capital sectors, respectively, and β ($0 < \beta < 1$) is the share of physical capital in the human capital sector.

We assume that Y^K is more intensive in physical capital ($\alpha > \beta$)⁶.

The planning problem is

$$\max_{C,v,u,K,H} \int_0^{+\infty} \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \quad (4)$$

subject to equations (2) and (3). $K_0 > 0$ and $H_0 > 0$ are given and $C \geq 0$. The parameter ρ represents the subjective discount rate. In Appendix A.1 we present the first order conditions of the model and the characterization of its transitional dynamics. In the next subsection we derive the analytical expression of the speed of convergence and the intuition associated with it.

2.2 The speed of convergence

We define P as the relative price of human capital in terms of goods and physical capital. That is, the relative price in this economy is given by the relative price of the two assets in the economy.

As shown in Appendix A.1, the differential equation of the relative price depends only on itself, making it possible to find a negative eigenvalue, driving the relative price to the steady state:

giving values to the parameters of the model. The former present an analytical and geometrical solution to the dynamics of the model.

⁶That is we assume the production of goods and physical capital is more intensive in physical capital. This leads to the stability of the differential equation of the relative price. The case where $\alpha < \beta$ is discussed in Appendix A.3.

$$\frac{\dot{P}}{P} = (r^K - \delta_K) - \left(\frac{1}{P} r^H - \delta_H \right) \quad (5)$$

The rental rate of physical and human capital are represented, respectively, by r^K and r^H . Assuming there is incomplete specialization in production, we show in Appendix A.1 that the rental rate of each capital is a function of the relative price:

$$r^K = \alpha A \phi^{\alpha-1} P^{\frac{\alpha-1}{\alpha-\beta}} \quad (6)$$

$$r^H = (1 - \alpha) A \phi^\alpha P^{\frac{\alpha}{\alpha-\beta}} \quad (7)$$

where $\phi \equiv \left[\frac{B}{A} \left(\frac{\beta}{\alpha} \right)^\beta \left(\frac{1-\alpha}{1-\beta} \right)^{\beta-1} \right]^{\frac{1}{\alpha-\beta}}$ is a constant.

The model has a balanced growth path. As is known, in the steady state $\frac{\dot{P}}{P} = 0$. Equation (5) gives the steady state value of the relative price P^* , as the rental rate of physical and human capital depends only on the relative price. Then we obtain the steady state rental rate of physical capital $(r^K)^*$ and human capital $(r^H)^*$ using equations (6) and (7), respectively. Notice that P^* , $(r^K)^*$ and $(r^H)^*$ only depend on technological parameters⁷.

Linearizing the system around the steady state gives the following equation to the relative price:

$$\dot{P} = a_{33} (P - P^*)$$

where a_{33} represents the partial derivative of the equation (5) with respect to P , at the steady state values. These derivations are in Appendix A.2.

All other variables of the system converge to the steady state following the saddle-path. Given the assumptions stated above, the dynamics of the system and its speed of convergence can be given by the coefficient a_{33} of the equation of the relative price:

$$a_{33} = - \frac{[(1 - \alpha) + \beta] (r^K)^* + \beta (\delta_H - \delta_K)}{\alpha - \beta}.$$

By a_{33} , we can see that (for a given ratio physical capital-human capital, k) an increase in the relative price, P , leads to an increase in the remuneration of the human capital, the factor used intensively in the production

⁷The steady state growth rate is equal to $g = \frac{1}{\sigma} [(r^K)^* - \delta_K - \rho] > 0$. See Appendix A.1.

of human capital ($\alpha > \beta$) and the latter expression is negative. This is in relation to the Stolper-Samuelson theorem. It follows that the relative price has a stabilizing effect in the transitional dynamics of the economy. On the other hand, the capital ratio has a destabilizing role. In Appendix A.2 we show that $a_{22} = \left(\frac{\partial y^K}{\partial k} - \frac{\partial y^H}{\partial k}\right) k^* > 0$. By a_{22} , we can see that (for a given relative price) an increase in the ratio physical capital-human capital, k , leads to an increase in the output of the consumption sector and a decrease in the output of the human capital sector and this expression is positive. This is in relation to the Rybczynski theorem.

For simplicity of the analysis, we assume $\delta_K = \delta_H$. The speed of convergence is given by⁸

$$\lambda_C = \frac{(1 - \alpha) + \beta}{\alpha - \beta} (r^K)^* \quad (8)$$

where $(r^K)^*$ is the value of the marginal productivity of capital in the steady state. This value depends only on technological parameters, as we can see looking at the differential equation of the relative price, leaving no role for the parameter of preferences. The speed of convergence increases with β and decreases with α . As the share of human capital in the production of human capital decreases ($1 - \beta$), decreasing returns of human capital in this sector set in faster. The same argument applies to α and decreasing returns of physical capital in the production sector of consumption goods and physical capital.

We can analyze the transitional dynamics of the system by taking into account the behavior of the relative price and the remuneration of the two capitals, as given by the non-arbitrage condition given by equation (5). Investing one unit of physical capital yields a net marginal product of physical capital in the consumption goods equal to $r^K - \delta_K$. On the other hand, investing $\frac{1}{P}$ units of human capital yields $\frac{P}{P} + \frac{1}{P}r^H - \delta_H$. In the steady state, $(r^K)^* = \left(\frac{1}{P}r^H\right)^* = r^* + \delta_K$, with $\delta_K = \delta_H$. We will call r^* the steady state interest rate. The speed of convergence and the transitional path apply to any situation around the steady state: $k_0 \neq k^*$. By assuming

⁸The papers referred to above, Bond et al. (1996) and Mulligan and Sala-i-Martin (1993), do not arrive at an expression for the speed of convergence. Using the Lucas (1988) model, Ortigueira and Santos (1997) present an expression for the speed of convergence, but the analytical solution is derived in the appendix of their paper. These authors also analyze the behaviour of the speed of convergence of the non-linearized model and they show that the linearized model represents well the behaviour of the transitional dynamics over substantial phases of the transition. Working with the linearized version of the transitional dynamics, our results are all derived analytically.

for example that $k_0 > k^*$, human capital is relatively less abundant and its remuneration will be relatively higher than physical capital: $\frac{1}{P}r^H > r^K$. Notice that there will be an incentive to invest relatively more in human capital. As $\alpha > \beta$ and the remunerations of the two capitals depend only on the relative price, taking into account equations (7) and (6), $\frac{1}{P}r^H$ will be decreasing and r^K will be increasing during the transition to the steady state. As human capital becomes relatively more available, the relative price will be decreasing.

Look at Appendix A.3 the case where $\alpha < \beta$.

We will return to these expressions of the speed of convergence when comparing the closed and open economies. The next Section will discuss the presence of adjustment costs in the closed economy and its effect in the speed of convergence.

3 The closed economy

Based on the model of the previous Section, now we introduce adjustment costs in the closed economy. Later we will compare this economy with an open economy using also adjustment costs.

3.1 The model

The planning problem is now

$$\max_{C,v,u,I^T,K,H} \int_0^{+\infty} \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \quad (9)$$

subject to

$$A(vK)^\alpha (uH)^{1-\alpha} = C + I^T \left[1 + m \left(\frac{I^T}{K} \right) \right] \quad (10a)$$

$$\dot{K} = I^T - \delta_K K \quad (10b)$$

$$\dot{H} = B[(1-v)K]^\beta [(1-u)H]^{1-\beta} - \delta_H H \quad (10c)$$

with K_0 and H_0 given. Equations (10a) and (10b) appear in the place of equation (2) of the model presented in Section 2. The expression of the

adjustment costs, $m\left(\frac{I^T}{K}\right)$, can be represented in the following way⁹:

$$m\left(\frac{I^T}{K}\right) = \frac{\frac{h}{2}\left(\frac{I^T}{K} - a\right)^2}{\frac{I^T}{K}}. \quad (11)$$

$$a = g + \delta_K$$

$$g = \frac{1}{\sigma} \left[(r^K)^* - \delta_K - \rho \right] > 0$$

The parameter h is the sensitivity of the adjustment costs to the ratio investment to physical capital. The steady state value of the Tobin's q will be $q^* = 1$ if we assume $a = g + \delta_K$, where g represents the steady state growth rate of the closed economy. This growth rate is equal to the steady state growth rate of the model of Section 2 and will also be equal to the steady state growth rate of the open economy. As we are using the same economic structure and all relevant variables have the same steady state values in both economies, we guarantee the comparability between the closed and the open economies.

The Hamiltonian for this problem is

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t} \left\{ A(vK)^\alpha (uH)^{1-\alpha} - C - I^T \left[1 + m\left(\frac{I^T}{K}\right) \right] \right\} + \eta q e^{-\rho t} (I^T - \delta_K K) + \mu e^{-\rho t} \left\{ B[(1-v)K]^\beta [(1-u)H]^{1-\beta} - \delta_H H \right\}$$

where η is the Lagrangian multiplier associated with equation (10a), ηq is the costate variable in installed physical capital¹⁰ and μ is the costate variable in human capital.

We obtain the following first order conditions¹¹:

$$C^{-\sigma} = \eta \quad (12a)$$

$$r^K = \alpha A(vK)^{\alpha-1} (uH)^{1-\alpha} = \frac{\mu}{\eta} \beta B [(1-v)K]^{\beta-1} [(1-u)H]^{1-\beta} \quad (12b)$$

⁹For this specification of adjustment costs see Ortigueira and Santos (1997), King and Rebelo (1993) and Summers (1981).

¹⁰Defining this costate variable as $\eta q e^{-\lambda t}$ rather than as a single variable is a matter of convenience, as it will become clear later on when we show that q plays a key role in determining investment.

¹¹Notice that equations (12a) to (12d) are equal to equations (A1) to (A4). We would like to point out that other properties of the model of Section 2, derived in Appendix A.1, apply in the closed economy with adjustment costs.

$$r^H = (1 - \alpha) A (vK)^\alpha (uH)^{-\alpha} = \frac{\mu}{\eta} (1 - \beta) B [(1 - v) K]^\beta [(1 - u) H]^{-\beta} \quad (12c)$$

$$\dot{\mu} = \mu \left(\rho + \delta_H - \frac{\eta}{\mu} r^H \right) \quad (12d)$$

$$q = 1 + h \left(\frac{I^T}{K} - a \right) \quad (12e)$$

$$\dot{q} = \left(\rho + \delta_K - \frac{\dot{\eta}}{\eta} \right) q - r^K - h \left(\frac{I^T}{K} - a \right) \frac{I^T}{K} + \frac{h}{2} \left(\frac{I^T}{K} - a \right)^2. \quad (12f)$$

The transversality conditions are:

$$\lim_{t \rightarrow +\infty} \eta q e^{-\rho t} K = 0 \text{ and } \lim_{t \rightarrow +\infty} \mu e^{-\rho t} H = 0.$$

Equation (12e) can be specified as

$$\frac{q-1}{h} + g = \frac{I^T}{K} - \delta_K = \frac{\dot{K}}{K}. \quad (13)$$

We can define $P = \frac{\mu}{\eta}$ as the relative price of human capital in terms of goods. Notice that we will be using now the marginal utility of consumption, as $\eta = C^{-\sigma} = U'(C)$. It follows that $\frac{\dot{P}}{P} = \frac{\dot{\mu}}{\mu} - \frac{\dot{\eta}}{\eta} = \frac{\dot{\mu}}{\mu} + \sigma \frac{\dot{C}}{C}$. We also define $c = \frac{C}{K}$, $k = \frac{K}{H}$, $y^K = \frac{Y^K}{K}$ and $y^H = \frac{Y^H}{H}$. As derived in Appendix A.1:

$$y^K \equiv \frac{Y^K}{K} = \frac{1}{\alpha - \beta} \left[(1 - \beta) r^K - \beta r^H \frac{1}{k} \right] \quad (14a)$$

$$y^H \equiv \frac{Y^H}{H} = \frac{1}{P} \frac{1}{\alpha - \beta} \left[-(1 - \alpha) r^K k + \alpha r^H \right]. \quad (14b)$$

The dynamic system is given by:

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \left(\frac{q-1}{h} + g \right) \quad (15a)$$

$$\frac{\dot{k}}{k} = \left(\frac{q-1}{h} + g \right) - (y^H - \delta_H) \quad (15b)$$

$$\dot{q} = \left(\rho + \delta_K + \sigma \frac{\dot{C}}{C} \right) q - r^K - (q-1)a - \frac{h}{2} \left(\frac{q-1}{h} \right)^2 \quad (15c)$$

$$\frac{\dot{P}}{P} = \left(\rho + \delta_H - \frac{r^H}{P} \right) + \sigma \frac{\dot{C}}{C} \quad (15d)$$

and it takes into account the transversality conditions for K and H .

The specificity of this model is that one needs to solve equation (15a). By using equation (10a) and the definition $c = \frac{C}{K}$, we have

$$c = y^K - \left(\frac{q-1}{h} + a \right) - \frac{h}{2} \left(\frac{q-1}{h} \right)^2 \quad (16)$$

and then c depends also on k , q and P . We can also specify equation (15a) as a function of the same variables. It follows that the differential equations of q and P will also be a function of the same variables. Notice that by using (16)

$$\frac{\dot{c}}{c} = \frac{\left(\dot{y}^K - \frac{q}{h} \dot{q} \right)}{c} \quad (17)$$

and by using equations (14a), (6) and (7):

$$\dot{y}^K = \frac{1}{\alpha - \beta} \left[(1 - \beta) \frac{\alpha - 1}{\alpha - \beta} r^K \frac{\dot{P}}{P} - \frac{\beta}{k} \frac{\alpha}{\alpha - \beta} r^H \frac{\dot{P}}{P} + \frac{\beta}{k} r^H \frac{\dot{k}}{k} \right]. \quad (18)$$

In this general form, the linearized system has three equations depending on three variables. With such a system, it is not possible to find a clear analytical solution for the dynamics of the economy. One expects that by increasing the adjustment costs the speed of convergence will decrease, as in the Solow model with adjustment costs¹². By assuming that human capital is produced only with human capital, Ortigueira and Santos (1997) make simulations of the speed of convergence for a closed economy with endogenous growth and adjustment costs for physical capital and find that by increasing the adjustment costs the speed of convergence decreases.

We will look now for conditions that give the possibility of finding an analytical solution for the speed of convergence of this model. In Section 5 we will see the behaviour of the general form of the model giving values to its parameters.

¹²For the one sector model, we can derive an explicit solution for the speed of convergence of the linearized system. Abel and Blanchard (1983) characterize geometrically the dynamics of the one sector model with adjustment costs.

3.2 The transitional dynamics and the speed of convergence

Analyzing the case where human capital is produced only with human capital ($\beta \rightarrow 0$), it is possible to characterize the transitional dynamics of the model through a system of the two differential equations on q and P . With $\beta \rightarrow 0$, equation (18) is given by:

$$\dot{y}^K = \frac{1}{\alpha} \frac{\alpha - 1}{\alpha} r^K \frac{\dot{P}}{P}. \quad (19)$$

In this case the production of tradable goods over physical capital and consumption over physical capital depend only on q and P . Then we can analyze the dynamics of the system with the differential equations of Tobin's q and of the relative price P . With $\beta \rightarrow 0$, it follows that $r^H = PB$, implying that $(r^K)^* = B = \left(\frac{r^H}{P}\right)^*$. Remember that we have assumed $\delta_K = \delta_H$.

The two differential equations to be considered for the transitional dynamics are now:

$$\dot{q} = \left(\rho + \delta_K + \sigma \frac{\dot{C}}{C} \right) q - r^K - (q - 1) a - \frac{h}{2} \left(\frac{q - 1}{h} \right)^2 \quad (20)$$

$$\frac{\dot{P}}{P} = (\rho + \delta_H - B) + \sigma \frac{\dot{C}}{C} \quad (21)$$

where

$$\frac{\dot{C}}{C} = \left(\frac{q - 1}{h} + g \right) + \frac{1}{c} \left(\frac{1}{\alpha} \frac{\alpha - 1}{\alpha} r^K \frac{\dot{P}}{P} - \frac{q}{h} \dot{q} \right). \quad (22)$$

By using equations (17) and (19) in equation (15a) we arrive at (22).

We would like to point out that c does not appear in the linearized version of equations (20) and (21). Notice that the constant associated with changes in c in equation (22) is equal to zero, as in the steady state $\dot{P} = \dot{q} = 0$.

For simplicity of the analysis and without loss of generality, we assume that $\sigma = 1$ and $\delta_K = \delta_H = 0$. An analytical solution for the speed of convergence of the linearized version of this model is derived and we can observe its behavior when changing the parameter of adjustment costs. The negative eigenvalue of the linearized system is derived in Appendix B.

Let $\lambda_{C(h)} = -\mu$ be the speed of convergence of this closed economy with

adjustment costs:

$$2\lambda_{C(h)} = -\rho + \left[\rho^2 + 4 \frac{1-\alpha}{\alpha} B \Delta \right]^{\frac{1}{2}} \quad (23)$$

where

$$\Delta = \frac{1}{h} A_p^* A_q^* = \frac{1}{h \left(1 + \frac{1}{c^*} \frac{1-\alpha}{\alpha} B \right) + \frac{1}{c^*}}.$$

In Appendix B we show that the speed of convergence of this closed economy is decreasing in the adjustment costs, that is $\frac{\partial \lambda_{C(h)}}{\partial h} < 0$. We also show that the expressions A_p^* and A_q^* are functions of the parameters and of the steady state values of the variables.

One would expect the speed of convergence of this economy to approach the speed of convergence of the closed economy presented in Section 2 when the adjustment costs go to zero (see equation (8) with $\beta = \delta_K = \delta_H = 0$)¹³. In fact:

$\lim_{h \rightarrow 0} \lambda_{C(h)} = \frac{1-\alpha}{\alpha} B$, corresponding to the speed of convergence of the same model of a closed economy without adjustment costs. That is $\lambda_C = \lambda_{C(h=0)}$.

It is also interesting to know what happens if the adjustment costs go to infinite. We have

$$\lim_{h \rightarrow \infty} \lambda_{C(h)} = 0.$$

We would like to point out one conclusion that will be used later on when comparing the speed of convergence of the closed and the open economies. Since the adjustment costs have a finite value, the speed of convergence of the closed economy is always greater than zero.

4 The open economy to capital mobility

First we want to point out that openness in this Section means access to international capital markets. We are not analyzing trade effects on growth and convergence.

We have seen that a two sector growth model for a closed economy presents a finite speed of convergence. Notwithstanding, the two sector open economy growth model based on the model of Section 2 does not have a finite speed of convergence, because the international interest rate ties down the marginal productivity of capital and, in this way, also determines

¹³Notice that in the general case equations (10a) and (10b) are reduced to equation (2), as the adjustment costs go to zero.

the steady state relative price. Thus, the stabilizing mechanism provided in the closed economy by the relative price when $\alpha > \beta$ is eliminated. Then the economy would jump immediately to the steady state¹⁴. We would like to point out that this situation happens because we are using only two factors of production¹⁵.

We develop an open economy model where physical capital and consumption will be tradable goods and human capital will be assumed nontradable. To avoid the problem of instantaneous adjustment, we will introduce adjustment costs in the accumulation of physical capital, as in the closed economy presented in Section 3. As the remuneration of physical capital is not restricted to be equal to the international interest rate during the transition, the stable mechanism of the relative price will be again at work as in Section 2.

Surprisingly, the speed of convergence of the open economy is smaller than the speed of convergence of the closed economy. We will see that an economy with incomplete specialization in production has a speed of convergence that does not depend on the adjustment costs. The speed of convergence also does not depend on the parameter of preferences in this open economy with adjustment costs.

These results are in contrast with the dynamics of the one sector open economy with adjustment costs.

¹⁴The Solow model predicts a finite speed of convergence for a closed economy. But a small open economy with the same characteristics would have an infinite speed of convergence. With perfect international capital mobility, the remuneration of capital in a small open economy must be equal to the international interest rate and capital flows would eliminate any difference in remunerations instantaneously. Even allowing for adjustment costs in the small open economy, the closed economy with the same structure has a smaller speed of convergence. Credit constraints can be introduced in an economy using physical and human capital in production, where only physical capital can be used as collateral for foreign borrowing. The previous conclusion does not change and the values of the speed of convergence in the model may be closer to the values one finds in the empirical literature. For a discussion on convergence of open economies see Barro and Sala-i-Martin (1995), chapter 3, and Obstfeld and Rogoff (1996), chapter 7.

¹⁵In a two-sector open economy exogenous growth model using physical capital, human capital and labor as input, one does not need to introduce adjustment costs for having transitional dynamics. Brock and Turnovsky (1994) develop a model with these elements and characterize its dynamics, but they were not looking for the speed of convergence of this open economy.

4.1 The model

The planning problem is now

$$\max_{C,v,u,I^T,K,H,B} \int_0^{+\infty} \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \quad (24)$$

subject to

$$\dot{D} = r^* D + A (vK)^\alpha (uH)^{1-\alpha} - C - I^T \left[1 + m \left(\frac{I^T}{K} \right) \right] \quad (25a)$$

$$\dot{K} = I^T - \delta_K K \quad (25b)$$

$$\dot{H} = B [(1-v)K]^\beta [(1-u)H]^{1-\beta} - \delta_H H \quad (25c)$$

with D_0 , K_0 and H_0 given.

Let D represent net foreign bonds accumulated by the economy and r^* be the international interest rate. Notice that equation (25a) replaces equation (10a) of the closed economy. The expression of the adjustment costs in equation (25a) is equal to the expression in equation (11) presented in the closed economy with adjustment costs. We are assuming that r^* is equal to $(r^K)^* - \delta_K$, as determined in the steady state of the closed economy. This implies that the closed and the open economy have the same production structure, as stated above, and that we are assuming the rest of the world is in the steady state. Notice also that both economies have the same growth rate in the steady state, as $g = \frac{1}{\sigma} [(r^K)^* - \delta_K - \rho] = \frac{1}{\sigma} (r^* - \rho)$.

The Hamiltonian for this problem is now

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t} \left\{ r^* D + A (vK)^\alpha (uH)^{1-\alpha} - C - I^T \left[1 + m \left(\frac{I^T}{K} \right) \right] \right\} + \\ + \eta q e^{-\rho t} (I^T - \delta_K K) + \mu e^{-\rho t} \left\{ B [(1-v)K]^\beta [(1-u)H]^{1-\beta} - \delta_H H \right\}$$

where η , ηq and μ are the costate variables in net foreign bonds, installed physical capital and human capital, respectively.

In addition to equations (12a) to (12e) of the closed economy, we obtain the following first order conditions¹⁶:

$$\dot{\eta} = \eta (\rho - r^*) \quad (26a)$$

¹⁶As for the closed economy with adjustment costs, the properties of the model of Section 2 also apply in the open economy.

$$\dot{q} = (r^* + \delta_K)q - r^K - h \left(\frac{I^T}{K} - a \right) \frac{I^T}{K} + \frac{h}{2} \left(\frac{I^T}{K} - a \right)^2. \quad (26b)$$

The transversality conditions are now:

$$\lim_{t \rightarrow +\infty} \eta q e^{-\rho t} K = 0, \quad \lim_{t \rightarrow +\infty} \mu e^{-\rho t} H = 0 \quad \text{and} \quad \lim_{t \rightarrow +\infty} \eta e^{-\rho t} B = 0.$$

As in the closed economy we have $P = \frac{\mu}{\eta}$, giving the relative price of human capital in terms of goods. In this open economy P is also the relative price of tradables over nontradables, that is a real exchange rate. As stated above, η is the value of foreign bonds. Remember that the costate variable of installed physical capital in the open economy is ηq , because of the adjustment costs as in the closed economy.

The dynamic system of this open economy is given by the following equations:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} (r^* - \rho) - \left(\frac{q-1}{h} + g \right) \quad (27a)$$

$$\dot{d} = \left[r^* - \left(\frac{q-1}{h} + g \right) \right] d + y^K - c - \left(\frac{q-1}{h} + a \right) - \frac{h}{2} \left(\frac{q-1}{h} \right)^2 \quad (27b)$$

$$\frac{\dot{k}}{k} = \left(\frac{q-1}{h} + g \right) - (y^H - \delta_H) \quad (27c)$$

$$\dot{q} = (r^* + \delta_K)q - r^K - (q-1)a - \frac{h}{2} \left(\frac{q-1}{h} \right)^2 \quad (27d)$$

$$\frac{\dot{P}}{P} = r^* - \left(\frac{r^H}{P} - \delta_H \right) \quad (27e)$$

and it takes into account the transversality conditions for D , K and H . We are representing $d = \frac{D}{K}$, implying $\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{K}}{K}$.

The steady state value of the relative price is given by equation (27e). Note that $q^* = 1$. With equation (27c) we have k^* . As we will see below, d^* and c^* depend on the initial conditions.

From the flow budget constraint, equation (25a), and by taking into account a no-Ponzi-game condition, we obtain the intertemporal budget constraint:

$$\int_0^{+\infty} C e^{-r^* t} = \int_0^{+\infty} \left\{ A (vK)^\alpha (uH)^{1-\alpha} - I^T \left[1 + m \left(\frac{I^T}{K} \right) \right] \right\} e^{-r^* t} + D_0$$

where $C_t = C_0 e^{\frac{1}{\sigma}(r^* - \rho)t}$.

4.2 The transitional dynamics and the speed of convergence

Consider now the equations of k , q and P . The linearized system of these three equations is given by

$$\begin{bmatrix} \dot{k} \\ \dot{q} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} * \begin{bmatrix} k - k^* \\ q - 1 \\ P - P^* \end{bmatrix}$$

where a_{ij} represents the partial derivative of the equation i with respect to the argument j , at the steady state values.

As in the closed economy, we consider the case where the share of physical capital in the tradable sector is greater than in the nontradable sector: $\alpha > \beta$. As the differential equation of the relative price is stable in this case, this condition leads to the stability and the dynamics of transition of the real exchange rate. Then¹⁷:

$$a_{11} = -\frac{\partial y^H}{\partial k} k^* > 0; a_{22} = r^* - g \text{ and } a_{33} = -\frac{\beta}{\alpha - \beta} (r^K)^* < 0.$$

The other coefficients are: $a_{12} = \frac{k^*}{h}$; $a_{13} = -\frac{\partial y^H}{\partial P} k^* > 0$; $a_{23} = -\frac{\partial r^K}{\partial P} > 0$.

Taking into account the discussion presented in Section 2, we see the coefficients a_{11} and a_{33} are related to the Rybczynski theorem and to the Stolper-Samuelson theorem, respectively. It also follows that the relative price has a stabilizing effect and the capital ratio a destabilizing role, in the transition dynamics of the economy.

The speed of convergence of this open economy, λ_O , is again determined with the differential equation of the relative price. Given the negative eigenvalue a_{33} , the expression of the speed of convergence is equal to

$$\lambda_O = \frac{\beta}{\alpha - \beta} (r^K)^*. \quad (28)$$

We want to point out that the speed of convergence of the open economy model does not depend on the adjustment costs. Since the economy is incompletely specialized, this result applies. This result stands in stark contrast with the small open economy neoclassical growth model with adjustment costs. The result is related to the fact that physical capital and

¹⁷In a_{33} we are assuming, as in the closed economy, that $\delta_K = \delta_H$.

human capital are complementary in both sectors of production and to the production of human capital being more intensive in human capital, the nontradable capital¹⁸.

We will provide more intuition on the behaviour of λ_O in Section 5, when comparing the closed and open economies.

Finally, we present the dynamics of foreign bonds. The details of the solution of equation (27b), linearized around the steady state, are in Appendix C. We have then

$$(d - d^*) = \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) e^{-\lambda_O t}. \quad (29)$$

where the signs of the expressions in Λ are: $y_P^K < 0$; $y_k^K > 0$; $\Pi > 0$; $\Gamma < 0$. It follows that the ambiguity of the sign of Λ comes from y_P^K being negative, as all other expressions are positive.

The ratio net foreign bonds over physical capital, d , is driven by the capital ratio, k . Because of the ambiguity of the sign of Λ , we have an ambiguity with the evolution of d . This ambiguity would disappear in the case $\alpha < \beta$, where the relative price has no transitional dynamics and the only term in Λ is $y_k^K < 0$.

Notice that we know d^* with this last equation, given $t = 0$ and a k_0 . Then one obtains c^* with equation (27b).

5 Comparing the speed of convergence of the closed and the open economies

This discussion on the speed of convergence proceeds in three steps. First, we assume small adjustment costs such that the closed economy can be represented by the structure without adjustment costs. In this case, we have shown that our closed economy approaches the model presented in Section 2. The intuition for the difference between the two speeds of convergence is presented. Secondly, we discuss the case with positive adjustment costs in both economies in a special case where the speed of convergence of the closed economy is also derived analytically. Finally, we consider the general case of the closed economy giving values to the parameters of the model.

¹⁸In Barro, Mankiw and Sala-i-Martin (1995) there is only one sector of production, where physical capital and human capital are complementary, but human capital cannot be used as collateral for foreign borrowing and then must be accumulated in the country. Notice that in Brock and Turnovsky (1994), a two-sector neoclassical growth model, there is no collateral nor adjustment costs.

5.1 The case where h is small

The speed of convergence of the open economy is smaller than the speed of convergence of the closed economy presented in Section 2. This main result of the paper can be clearly read in the expressions of the speed of convergence of the closed and the open economy, λ_C and λ_O , given by equations (8) and (28) respectively. The difference is the expression $(1 - \alpha)$ in λ_C . As far as $0 < \alpha < 1$, the remuneration of physical capital will be changing during the transitional path in the closed economy. In these conditions, the gap between remunerations will be bigger in the closed economy than in the open economy. Only if $\alpha = 1$ would this effect disappear, as $(1 - \alpha) = 0$. By equation (6), the remuneration of physical capital would be $r^K = \alpha A$.

We develop now some intuition on the transitional dynamics of the relative prices in the open economy. As in Section 2 with equation (5), relevant information is provided by equation (27e). The relative incentive to accumulate one or another capital depends on their remuneration. Consider again the case $k_0 > k^*$ ¹⁹. Human capital is relatively less abundant and its remuneration will be relatively higher than physical capital: $\frac{1}{P}r^H - \delta_H > r^*$. Notice that now there is an alternative of investment given by the international interest rate r^* . This interest rate is fixed during the transition, instead of changing like the remuneration of physical capital in the closed economy where $r^K - \delta_K < r^*$. This differentiated pattern in the closed and the open economies leads to a gap between remunerations which is smaller in the open economy. It follows that the incentive to invest relatively more in human capital is bigger in the closed economy and this economy will be going faster to the steady state than the open economy. In this economy with two capitals produced with specific technologies, the speed of convergence is smaller in the open economy than in the closed economy.

Let assume $\alpha < \beta$. With the production of goods being more intensive in human capital, it follows that the negative eigenvalue would be given by $a_{11} = -\frac{\partial y^H}{\partial k}k^*$. The parameter of preferences does not have influence on the speed of convergence, although we are considering an open economy with adjustment costs. These conclusions are the opposite of what we find using an open economy with one sector and adjustment costs. By taking into account the corresponding expression for the closed economy, as presented in Appendix A.3, the speed of convergence is also smaller in the open economy than in the closed economy. With $\alpha < \beta$, we have $\lambda_O = \frac{\partial y^H}{\partial k}k^*$ and $\lambda_C = \frac{\partial y^H}{\partial k}k^* (1 + \xi)$, where $\xi > 0$.

¹⁹The conclusions do not change if we consider instead the case $k_0 < k^*$.

5.2 The case where β is small

Now we compare the speed of convergence in the closed and the open economies, both with adjustment costs. We would like to point out that the intuition of our main result was derived above in an environment where there are adjustment costs in the open economy, but these adjustment costs were not present in the closed economy of Section 2. At least for some measure of adjustment costs in the closed economy the result presented is applicable.

As stated previously, we want to keep all the results derived analytically. We have shown in Section 3 that the speed of convergence of the closed economy with adjustment costs could be derived analytically if $\beta \rightarrow 0$. This is the case of human capital being produced only with human capital, as in Lucas (1988). It follows that we have a special case in the open economy, as its speed of convergence will go to zero (see equation (28) with $\beta \rightarrow 0$ and $\delta_K = \delta_H = 0$). In this case, with $\beta \rightarrow 0$ and taking into account equation (23), the speed of convergence of the closed economy is always higher than the speed of convergence of the open economy.

5.3 The general case giving values to the parameters of the model

To explore by how much the speed of convergence of the closed economy is affected by the adjustment costs, now we carry out simulations, using the following values²⁰:

- steady state growth rate: $g = 0.02$
- rate of depreciation: $\delta_K = \delta_H = 0.05$
- subjective preference rate: $\rho = 0.03$
- (inverse) intertemporal elasticity of substitution: $\sigma = 1.5$
- share of physical capital in production of goods: $\alpha = 0.4$
- steady state interest rate: $r^* = 0.06$
- level of technology: $B = 0.11$ $A = 1.516$

²⁰These values of the parameters are in line with the literature. In the general model Barro and Sala-i-Martin (1995) use $\beta < 0.4$. Most of the literature uses the Lucas (1988) where $\beta = 0$. See also Ortigueira and Santos (1997).

Table 1: Comparing the speed of convergence of closed and open economies

	λ_C					λ_O
	$h = 0$	$h = 5$	$h = 10$	$h = 16$	$h = 32$	$h > 0$
$\beta = 0$	0.165	0.1175	0.1034	0.0951	0.0854	0
$\beta = 0.1$	0.25(6)	0.1456	0.122	0.1089	0.094	0.03(6)

Notes: λ_C and λ_O represent the speed of convergence of the closed and open economy, respectively. h is the adjustment cost and β is the share of physical capital in the production of human capital. See the text for the values of other parameters.

We set $B = 0.11$ and A is adjusted so as to maintain the steady state interest and growth rates. Notice that using $\beta = 0$ and taking into account equation (5) in the steady state, $B = (\frac{1}{P}r^H)^* = (r^K)^* = r^* + \delta = 0.11^{21}$.

All the results are presented in Table 1. Remember that the expressions for $\lambda_C(h = 0)$ and for λ_O are given by equations (8) and (28), respectively. We simulate the values of λ_C for $h > 0$ with MATLAB²².

As we have seen analytically with $\beta = 0$, the speed of convergence is always higher in the closed economy than in the open economy. We confirm these results for the range of values considered for the adjustment costs.

Setting $\beta = 0.1$ in the general case, we confirm our analytical result for any of the values considered for the adjustment costs²³. That is, the value of λ_C is greater than that of λ_O .

After providing the intuition of the main result of the paper in the first step of this discussion, we have shown above that our main conclusion applies using two economies (closed and open) with adjustment costs. The results for the speed of convergence of closed and open economies with two capitals produced with specific technologies are opposite to the results of the one

²¹Notice that in this case A does not affect $(r^K)^*$, as $B = (\frac{1}{P}r^H)^* = (r^K)^*$. Things are different with $\beta > 0$.

²²The values of h are in line with those presented by Ortigueira and Santos (1997). In their paper leisure enters the utility function and has, together with h , an effect on the speed of convergence.

²³In this case we set $A = 1.516$.

sector growth model.

6 Conclusion

We developed an endogenous growth model with physical capital and human capital to compare the speed of convergence of open and closed economies. As human capital was nontradable in the open economy, partial capital mobility was the only difference between these two economies. It was possible to analyze the transitional dynamics of the model with a non-arbitrage condition. The difference between remuneration of the capitals gives the incentive to accumulate relatively more in one of the capitals and this condition only depends on the relative price. The main implication of the model was a speed of convergence smaller for the open economy, a result in sharp contrast with the one sector growth model.

In terms of policy implications, our study suggests more reflection on the idea that capital mobility will increase the convergence of economies to their steady state. Given the implication of the model it would be of interest to analyze empirically these issues. The empirical literature, although looking for proxies of financial openness and their effect on growth through alternative specifications, has not analyzed in a systematic way the effects of capital mobility on the speed of convergence. Further empirical studies on this issue seems to be an interesting topic for future research.

Appendix A Derivations for the model of Section 2

In this Appendix we provide derivations for Section 2.

A.1 First order conditions

The Hamiltonian for the problem in Section 2 is given by

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t} \left[A(vK)^\alpha (uH)^{1-\alpha} - \delta_K K - C \right] + \mu e^{-\rho t} \left\{ B[(1-v)K]^\beta [(1-u)H]^{1-\beta} - \delta_H H \right\}$$

where η and μ are the costate variables for physical and human capital, respectively.

We obtain the following first order conditions:

$$C^{-\sigma} = \eta \tag{A1}$$

$$r^K = \alpha A(vK)^{\alpha-1} (uH)^{1-\alpha} = \frac{\mu}{\eta} \beta B[(1-v)K]^{\beta-1} [(1-u)H]^{1-\beta} \tag{A2}$$

$$r^H = (1-\alpha) A(vK)^\alpha (uH)^{-\alpha} = \frac{\mu}{\eta} (1-\beta) B[(1-v)K]^\beta [(1-u)H]^{-\beta} \tag{A3}$$

$$\dot{\mu} = \mu \left(\rho + \delta_H - \frac{\eta}{\mu} r^H \right) \tag{A4}$$

$$\dot{\eta} = \eta (\rho + \delta_K - r^K) \tag{A5}$$

The transversality conditions are:

$$\lim_{t \rightarrow +\infty} \eta e^{-\rho t} K = 0 \text{ and } \lim_{t \rightarrow +\infty} \mu e^{-\rho t} H = 0.$$

Let define $P = \frac{\mu}{\eta}$ as the relative price of human capital in terms of goods. The costate variables represent the value of one unit of each capital.

It follows that $\frac{\dot{P}}{P} = \frac{\dot{\mu}}{\mu} - \frac{\dot{\eta}}{\eta}$.

Using equations (A2) and (A3), we show that the physical capital-human capital ratio in each sector of production is a function only of the relative price. It follows that the rental rate of each capital is also a function of the

relative price. Assuming there is incomplete specialization in production:

$$k^K \equiv \frac{vK}{uH} = P^{\frac{1}{\alpha-\beta}} \phi \quad (\text{A6})$$

$$k^H \equiv \frac{(1-v)K}{(1-u)H} = \frac{1-\alpha}{1-\beta} \frac{\beta}{\alpha} P^{\frac{1}{\alpha-\beta}} \phi \quad (\text{A7})$$

$$r^K = \alpha A \phi^{\alpha-1} P^{\frac{\alpha-1}{\alpha-\beta}} \quad (\text{A8})$$

$$r^H = (1-\alpha) A \phi^\alpha P^{\frac{\alpha}{\alpha-\beta}} \quad (\text{A9})$$

where

$$\phi \equiv \left[\frac{B}{A} \left(\frac{\beta}{\alpha} \right)^\beta \left(\frac{1-\alpha}{1-\beta} \right)^{\beta-1} \right]^{\frac{1}{\alpha-\beta}}. \quad (\text{A10})$$

The capital ratio in the consumption and human capital sector are, respectively, k^K and k^H . The rental rate of physical and human capital are represented, respectively, by r^K and r^H . We also define $c = \frac{C}{K}$ and $k = \frac{K}{H}$.

With full employment of factors and the definitions of k^K and k^H given by the equations (A6) and (A7), there is a relation ($uk^K + (1-u)k^H = k$) that gives the fraction of physical capital used in the goods sector, u , as a function of the physical capital-human capital ratio of the economy, k , and the relative price, P :

$$u = \frac{k - k^H}{k^K - k^H} = u(k, P). \quad (\text{A11})$$

Using these previous equations, the production functions can be expressed as a function of the relative price and the capital ratio:

$$y^K \equiv \frac{Y^K}{K} = \frac{1}{\alpha-\beta} \left[(1-\beta) r^K - \beta r^H \frac{1}{k} \right] \quad (\text{A12})$$

$$y^H \equiv \frac{Y^H}{H} = \frac{1}{P} \frac{1}{\alpha-\beta} \left[-(1-\alpha) r^K k + \alpha r^H \right]. \quad (\text{A13})$$

The dynamic system is written as:

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \frac{1}{\sigma} (r^K - \delta_K - \rho) - (y^K - \delta_K - c) \quad (\text{A14})$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{H}}{H} = (y^K - \delta_K - c) - (y^H - \delta_H) \quad (\text{A15})$$

$$\frac{\dot{P}}{P} = (r^K - \delta_K) - \left(\frac{1}{P} r^H - \delta_H \right) \quad (\text{A16})$$

and takes into account the transversality conditions for K and H .

The model has a balanced growth path. In the steady state $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{P}}{P} = 0$. Equation (5) gives the steady state value of the relative price P^* , as the rental rate of physical and human capital depends only on the relative price. We obtain the steady state rental rate of physical capital $(r^K)^*$ and human capital $(r^H)^*$ using equations (A8) and (A9), respectively. The steady state growth rate is given by $g = \frac{1}{\sigma} [(r^K)^* - \delta_K - \rho] > 0$. Then we obtain the steady state ratio of capitals (k^*) and consumption over physical capital (c^*) using equations (A14) and (A15).

A.2 The transitional dynamics and the speed of convergence

We assume that the physical sector is more intensive in physical capital ($\alpha > \beta$) and that there is incomplete specialization in production.

Linearizing the system around the steady state gives:

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} * \begin{bmatrix} c - c^* \\ k - k^* \\ P - P^* \end{bmatrix}$$

where a_{ij} represents the partial derivative of the equation i with respect to the argument j , at the steady state values. $a_{11} = c^*$; $a_{12} = -\frac{\partial y^K}{\partial k} c^* < 0$; $a_{21} = -k^*$; $a_{22} = \left(\frac{\partial y^K}{\partial k} - \frac{\partial y^H}{\partial k} \right) k^* > 0$. The expressions of the other coefficients are: $a_{13} = \left(\frac{1}{\sigma} \frac{\partial r^K}{\partial P} - \frac{\partial y^K}{\partial P} \right) c^*$, which has an ambiguous sign; $a_{23} = \left(\frac{\partial y^K}{\partial P} - \frac{\partial y^H}{\partial P} \right) k^* < 0$. To this sub-matrix, the trace is $a_{11} + a_{22} > 0$ and the determinant is equal to $-\frac{\partial y^H}{\partial k} k^* c^* > 0$.

The system has one negative eigenvalue (for the capital ratio k) and two positive eigenvalues (for the control variables P and c). Then the dynamics of the system and its speed of convergence can be given by the coefficient a_{33} of the equation of the relative price:

$$a_{33} = -\frac{[(1 - \alpha) + \beta] (r^K)^* + \beta (\delta_H - \delta_K)}{\alpha - \beta}.$$

Assuming $\delta_K = \delta_H$, the speed of convergence is given by

$$\lambda_C = \frac{(1 - \alpha) + \beta}{\alpha - \beta} (r^K)^* . \quad (\text{A17})$$

A.3 The case where the production of human capital is more intensive in physical capital ($\alpha < \beta$)

We briefly analyze this case. Now $a_{33} > 0$ and the negative eigenvalue will be found in a two-by-two matrix: the relative price mechanism is unstable (for a given k) and the quantities mechanism is stable (for a given relative price).

The trace is

$$\text{Tr} = a_{11} + a_{22} = c^* + \left(\frac{\partial y^K}{\partial k} - \frac{\partial y^H}{\partial k} \right) k^* .$$

And the determinant is given by

$$\text{Det} = a_{11}a_{22} - a_{12}a_{21} = -c^* \frac{\partial y^H}{\partial k} k^* .$$

The negative eigenvalue is equal to

$$2\mu = \text{Tr} - \sqrt{\text{Tr}^2 - 4\text{Det}} ,$$

where

$$\sqrt{\text{Tr}^2 - 4\text{Det}} = \left[c^* + \left(\frac{\partial y^K}{\partial k} + \frac{\partial y^H}{\partial k} \right) k^* \right]^2 \underbrace{- 4 \frac{\partial y^K}{\partial k} \frac{\partial y^H}{\partial k} k^*}_{>0} .$$

In this case the speed of convergence is given by

$$\lambda_C = -\mu = \frac{\partial y^H}{\partial k} k^* (1 + \xi) \quad (\text{A18})$$

with $\xi > 0$.

Appendix B The negative eigenvalue associated with the closed economy

Here we derive the negative eigenvalue associated with the linearization of the system given by equations (20), (21) and (22) in Section 3.

When human capital is produced only with human capital ($\beta \rightarrow 0$), the system is analyzed with the two differential equations on q and P . As presented in the text, the system is given by:

$$\dot{q} = \left(\rho + \delta_K + \sigma \frac{\dot{C}}{C} \right) q - r^K - (q - 1) a - \frac{h}{2} \left(\frac{q - 1}{h} \right)^2$$

$$\frac{\dot{P}}{P} = (\rho + \delta_H - B) + \sigma \frac{\dot{C}}{C}$$

where

$$\frac{\dot{C}}{C} = \left(\frac{q-1}{h} + g \right) + \frac{1}{c} \left(\frac{1-\alpha}{\alpha} r^K \frac{\dot{P}}{P} - \frac{q}{h} \dot{q} \right).$$

We would like to point out that in the linearization of the equation above, the constant associated with changes in c is equal to zero as \dot{P} and \dot{q} are also equal to zero in the steady state.

Reorganizing terms leads to

$$\dot{q} = A_q \left[(\rho + \delta_K) q - r^K - (q-1) a - \frac{h}{2} \left(\frac{q-1}{h} \right)^2 + q A_P \sigma \frac{q-1}{h} + q g \sigma \right]$$

$$\frac{\dot{P}}{P} = A_P \left[(\rho + \delta_H - B) + \sigma \left(\frac{q-1}{h} + g \right) - \frac{\sigma}{c} \frac{q}{h} \dot{q} \right]$$

where

$$A_P = \frac{1}{1 + \frac{\sigma}{c} \frac{1-\alpha}{\alpha} r^K}$$

and

$$A_q = \frac{1}{1 + q A_P \frac{\sigma}{c} \frac{q}{h}}.$$

The linearized system can be represented by:

$$\begin{bmatrix} \dot{q} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} * \begin{bmatrix} q-1 \\ P - P^* \end{bmatrix}$$

where

$$a_{11} = \left[\rho + g(\sigma - 1) + A_P^* \frac{\sigma}{h} \right] A_q^*$$

$$a_{12} = \frac{1-\alpha}{\alpha} \left(\frac{r^K}{P} \right)^* A_q^*$$

$$a_{21} = A_P^* P^* \left[\frac{\sigma}{h} - \frac{\sigma}{c^*} \frac{1}{h} A_q^* \left(\rho + g(\sigma - 1) + A_P^* \frac{\sigma}{h} \right) \right]$$

$$a_{22} = -A_P^* A_q^* \frac{\sigma}{c^*} \frac{1}{h} \frac{1-\alpha}{\alpha} (r^K)^*.$$

Notice that A_P^* and A_q^* correspond to the expression of A_P and A_q , but are evaluated at the steady state values of the variables of the model.

Without loss of generality, we assume that $\sigma = 1$ and $\delta_K = \delta_H = 0$, and we have an analytical solution for the speed of convergence of this model.

Now $a = g = B - \rho$ and using equation (10a) one can show that $c^* = \frac{(r^K)^*}{\alpha} - g = \frac{1-\alpha}{\alpha}B - \rho$. It follows that:

- the trace is equal to $\text{Tr} = \rho$
- the determinant equal to $\text{Det} = -\frac{1-\alpha}{\alpha}B\frac{1}{h}A_P^*A_q^*$
- and the negative eigenvalue equal to $2\mu = \rho - \sqrt{\rho^2 + 4\frac{1-\alpha}{\alpha}B\frac{1}{h}A_P^*A_q^*}$.

Let $\lambda_{C(h)} = -\mu$ be the speed of convergence of this closed economy with adjustment costs. By rearranging terms, we can represent the speed of convergence as

$$2\lambda_{C(h)} = -\rho + \left[\rho^2 + 4\frac{1-\alpha}{\alpha}B\Delta \right]^{\frac{1}{2}} \quad (\text{A19})$$

where

$$\Delta = \frac{1}{h}A_P^*A_q^* = \frac{1}{h\left(1 + \frac{1}{c^*}\frac{1-\alpha}{\alpha}B\right) + \frac{1}{c^*}}.$$

The expression of the speed of convergence corresponds to equation (23) in the text.

The speed of convergence of this closed economy is decreasing in the adjustment costs:

$$\begin{aligned} \frac{\partial \lambda_{C(h)}}{\partial h} &= \frac{1}{2} \left[\rho^2 + 4\frac{1-\alpha}{\alpha}B\Delta \right]^{\frac{1}{2}-1} * \\ &* 4\frac{1-\alpha}{\alpha}B\Delta^2 \left(1 + \frac{1}{c^*}\frac{1-\alpha}{\alpha}B \right) (-1) < 0. \end{aligned} \quad (\text{A20})$$

Appendix C The dynamics of foreign bonds

This appendix derives the dynamics of foreign bonds presented in Section 4.

We can now analyze the dynamics of consumption and of net foreign bonds. Linearizing equation (27a) we have

$$\dot{c} = -\frac{c^*}{h}(q-1) = -\frac{c^*}{h}\Gamma(k_0 - k^*)e^{-\lambda_O t}$$

and the solution of this differential equation is given by

$$(c - c^*) = \frac{-\frac{c^*}{h}\Gamma}{-\lambda_O} (k_0 - k^*) e^{-\lambda_O t}.$$

By linearizing equation (27b) and by taking into account the previous equations:

$$\dot{d} = (r^* - g)(d - d^*) + \Lambda(k - k^*)$$

with

$$\Lambda = \left[(y_P^K \Pi + y_k^K) + \frac{c^* \Gamma}{- \lambda_O} - \left(\frac{d^* + 1}{h} \right) \Gamma \right]$$

and

$$\Gamma = \frac{-(\lambda_O + a_{11})}{\frac{k^*}{h} - \frac{a_{13}}{a_{23}}(r^* - g + \lambda_O)} \text{ and } \Pi = \frac{-(r^* - g + \lambda_O)}{a_{23}} \Gamma.$$

The signs of the expressions in Λ are: $y_P^K < 0$; $y_k^K > 0$; $\Pi > 0$; $\Gamma < 0$. These expressions are related to the solution of the system of differential equations, k , q and P . It follows that the ambiguity of the sign of Λ comes from y_P^K being negative, as all other expressions are positive.

The solution of this differential equation is given by:

$$(d - d^*) = \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) e^{-\lambda_O t} + \left[d_0 - d^* - \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) \right] e^{(r^* - g)t} \quad (\text{A21})$$

with the equation in brackets set equal to zero to observe the transversality condition of net foreign bonds, D . We have then

$$(d - d^*) = \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) e^{-\lambda_O t}. \quad (\text{A22})$$

This is equation (29) presented in the text.

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