Walrasian prices in a market with consumption rights

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Literature and overview

- Tradable - licence systems are the focus of current interest in market-based environmental policy
- A tradable licence system confers the agents holding it the right to consume. Some examples are the right to capture a protected species of fish or the right to emit pollutants at a certain rate
- In a market system these licences should be tradable and the desirable rule governing exchange of emission rights should be based on a market-price system

Literature and overview

- A precise formulation of this kind of model appears in the seminal paper by **Montgomery (JET, 1972)**:

  > *In a scenario where an exchange of such licences between polluters at different locations is considered, Montgomery shows that market equilibrium in emission licences exists and that with some restrictions on the initial allocation of licences, the market equilibrium with emission licenses is efficient.*

Literature and overview

- The subsequent *environmental markets,* thus introduced, appears as an alternative way to control emission via regulatory designs (see **Baumol and Oates (1988)** and **Montero (RAND, 2001)**).

- On the other hand **Boyd and Conley (JET,1997)** were the first to treat directly the efficiency problem in presence of externalities, opposed to an indirectly way through *Arrovian* commodities, arguing that the essential non-convexities highlighted by **Starrett (JET,1992)** are due to unboundedness of the negative effects rather than the externalities themselves.

Literature and overview

- Later, **Conley and Smith (JME,2005)** extended the Boyd and Conley model to allow firms to benefit from public goods and be damaged by externalities, proving the existence and Pareto optimality of a competitive equilibrium.
- More recently, **Mandel (ET,2009)** focuses on the influence on the general equilibrium of an economy of the opening of markets of allowances. Assuming the existence of an equilibrium before the opening of allowances' markets, Mandel describes the changes in the firms behavior which guarantee that an equilibrium can be reached in the enlarged economy.

Literature and overview

- In our model, we consider a scenario in which limits to the consumption of certain commodities have been established. The consumption of these commodities requires the availability of certain rights or licences for its consumption as well (**consumption rights**)
- We work in an exchange economy with externalities, where there is an external restriction for the consumption of goods
  - (-) limits to the consumption of certain raw materials have been established in order to restrict the potential negative effects which this consumption produces.

Literature and overview

- Restrictions have consequences on the individual's budget constraint:
  - (-) agents can not consume certain quantities of specific commodities. Even an agent may not consume the totality of her endowment.
  - (-) it may affect the agents budget sets, since in order to consume they will need to have the required rights
  - (-) consumption rights may be traded in the market (buy or sell)
- Consumption rights do not participate in preferences.

Literature and overview

Our approach differs from those by **Boyd and Conley**, **Conley and Smith** and **Mandel** *op.cit.* in the following aspects:

(i) We suppose the existence of an external function which evaluates the potential negative effects derived from each contract. This function associates to every consumption plan (contract) a theoretical amount of rights of each type

(ii) We do not consider production in our setting explicitly

(iii) In our model, agents evaluate their utility considering all the consequences involved in their consumption plan

Literature and overview

(iv) Our model is a pure exchange market in which consumption rights must be required at the same time as buying contracts are signed for raw materials (consumption goods)

(v) The model does not require to measure the actual negative effects of consumption.

(vi) We do not introduce new goods and markets (*Arrovian commodities*)

The model

[1] Exchange economy with externalities in consumption

- ℓ consumption goods
- m consumers indexed by $I = \{1, \ldots, m\}$
- Consumption set: $X_i \subset \mathbb{R}^\ell_+$ $\left(X = \prod_{i \in I} X_i,\ X_{-i} = \prod_{j \neq i} X_j\right)$
- Endowments: $\omega_i \in X_i,\ i \in I$
- Preferences:

$$u_i : X_{-i} \times X_i \to \mathbb{R}$$

[2] Cap-setting Process (CSP)

- Limits to consumption are given by the mapping

$$f : \mathbb{R}^{\ell}_{+} \to \mathbb{R}^{k}_{+}$$

- For $j \in K = \{1, \ldots, k\}$, the **CSP sets a limit** $R_j \in \mathbb{R}_+$ on the total allocation:

$$for\ (x_i) \in X,\ \sum_{i \in I} f_j(x_i) \leq R_j$$

- For $j \in K$ there is **consumption right** (or licence) and each individual $i \in I$ is endowed with an amount of each of them:

$$r_i = (r_i^j) \in \mathbb{R}^k_+\ s.t.\ \sum_{i \in I} r_i^j = R_j,\ j \in K.$$

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[3] Consumption rights are tradable

- If agent $i \in I$ decides to consume $x \in X_i$ then she must have an amount $f(x) \in \mathbb{R}^K_+$ of **consumption rights, that can be traded in the market**
- $r^j_i - f_j(x) < 0 \Rightarrow$ buy consumption rights type $j \in K$ in order to consume $x \in X_i$ (> 0 may sell it)
- For a price $s \in \mathbb{R}^k_+$ for right, the consumption of x implies that the total wealth she can obtain (or pay if negative) from this side is

$$s \cdot [r_i - f(x)] \in \mathbb{R}$$

Budget set

Given $(p, s) \in \Delta$, the budgetary set for individual $i \in I$ is defined by

$$B_i(p, s) = \{x \in X_i \mid p \cdot x \leq p \cdot \omega_i + s \cdot [r_i - f(x)]\}.$$

Economy with consumption rights

An economy with consumption rights is defined as

$$\mathcal{E}_r = ((u_i)_{i \in I}, (\omega_i)_{i \in I}, (r_i)_{i \in I}, f).$$

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Feasible allocations

We say that $(x_i) \in X = \prod_{i \in I} X_i$ is a feasible allocation for the economy $\mathcal{E}_r$ if the following holds true

(a) $\sum_{i \in I} x_i \leq_\ell \omega = \sum_{i \in I} \omega_i \in \mathbb{R}^\ell_+$.

(b) $\sum_{i \in I} f(x_i) \leq_k R = \sum_{i \in I} r_i \in \mathbb{R}^k_+$.

The set of feasible allocation for the economy $\mathcal{E}_r$ is denoted by $\mathcal{F}_r$.

A remark

Note that ω does not necessarily belong to $\mathcal{F}_r$.

Moreover, for some prices s of the licences and for a given individual $i \in I$, her endowments ω_i do not necessarily belong to the budgetary set

$$B_i(p, s) = \{x \in X_i \mid p \cdot x \leq p \cdot \omega_i + s \cdot [r_i - f(x)]\} .$$

Nash-Walras equilibrium

We say that $((p^*, s^*), (x_i^*)) \in \Delta \times \mathbb{R}_+^{m\ell}$ is a Nash-Walras equilibrium for the economy $\mathcal{E}_r$ if

(a) $(x_i^*) = (x_{-i}^*, x_i^*) \in \mathcal{F}_r$

(b) for each $i \in I$, $x_i^* \in B_i(p^*, s^*)$ and x_i^* maximizes $u_i(x_{-i}^*, .)$ on $B_i(p^*, s^*)$.

Walras' Law

Lemma 1

Suppose that $f : \mathbb{R}^{\ell}_{+} \to \mathbb{R}^{k}_{+}$ is continuos and that for $i \in I$ and for any $x_{-i} \in X_{-i}$, $u_i(x_{-i}, \cdot) : X_i \to \mathbb{R}$ is locally non-satiated. Given $((p^*, s^*), (x_i^*))$ a Nash-Walras equilibrium of $\mathcal{E}_r$, if for $x_i \in X_i$ holds that $u_i(x^*) \leq u_i(x^*_{-i}, x_i)$, then

$$p^* \cdot x_i \geq p^* \cdot \omega_i + s^* \cdot [r_i - f(x_i)] \, .$$

Walras' Law

Under the conditions of Lemma 1, if $((p^*, s^*), (x_i^*))$ is a Nash-Walras equilibrium of $\mathcal{E}_r$ then

$$p^* \cdot \left[\sum_{i \in I} x_i^* - \omega \right] = 0, \;\; s^* \cdot \left[\sum_{i \in I} f(x_i^*) - R \right] = 0.$$

Walras' Law

Consequence of Walras' Law

An effective cap on a commodity t

$$(\sum_{i \in I} x_i^* - \omega)_t < 0$$

implies that the equilibrium price of that commodity p_t is zero and that the price of the consumption right becomes the relevant price

Existence of a Nash-Walras Equilibrium

Assumptions

- **Assumption C.** For each $i \in I$, $X_i \subseteq \mathbb{R}^{\ell}_{+}$ is convex, closed and $0_{\ell},\ \omega_i \in X_i$.
- **Assumption S.** For each $i \in I$, $\omega_i \in \mathbb{R}^{\ell}_{++}$ and $r_i \in \mathbb{R}^{k}_{++}$.
- **Assumption R.** $f_j : \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ is convex, continuous and $f_j(0) = 0$, for all $j = 1, ..., K$
- **Assumption U.** For each $i \in I$, the utility function u_i is continuous and, given any $x_{-i} \in X_{-i}$, the function $u_i(x_{-i}, .) : X_i \to \mathbb{R}$ is locally non-satiated and quasi-concave.

Auxiliary economy

$\mathcal{E}_r^M$ differs from $\mathcal{E}_r$ only in the consumption sets:

$$X_i^M = X_i \cap clB\left(0_\ell, M\|\omega\|\right), \quad M > 1.$$

Lemma 2

Under assumptions **C**, **S** and **R**, the correspondence $B_i^M : \Delta \to X_i^M$ s.t

$$B_i^M(p,s) = \left\{x \in X_i^M \mid p \cdot x \leq p \cdot \omega_i + s \cdot [r_i - f(x)]\right\}$$

is upper and lower semi-continuous and compact and convex valued.

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Theorem: existence of equilibrium

Under assumptions **C**, **S**, **R** and **U** there exist a Nash-Walras equilibrium for the economy $\mathcal{E}_r$.

- **Weak efficiency:** in order to show a kind of First Welfare Theorem (FWT) in our model, we will consider a particular definition of optimality.

Nash Pareto

We say that $x^* = (x_i^*) \in \mathcal{F}_r$ is a **Nash-Pareto optimum** for the economy $\mathcal{E}_r$ if there does not exist another feasible allocation $(x_i') \in \mathcal{F}_r$ such that for each $i \in I$, $u_i(x^*) \leq u_i(x_{-i}^*, x_i')$ and for some $i_0 \in I$, $u_{i_0}(x^*) < u_{i_0}(x_{-i_0}^*, x_{i_0}')$.

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Theorem: First Welfare Theorem

Suppose that $f : \mathbb{R}^{\ell}_{+} \to \mathbb{R}^{k}_{+}$ is continuous and that for $i \in I$ and for any $x_{-i} \in X_{-i}$, $u_i(x_{-i}, \cdot) : X_i \to \mathbb{R}$ is locally non-satiated. If $((p^*, s^*), (x_i^*))$ is an equilibrium of $\mathcal{E}_r$, then (x_i^*) is a Nash-Pareto optimum for the economy $\mathcal{E}_r$.

Main conclusions and further work

1. This paper deals with the problem of setting a price system for licences or consumption rights in an exchange economy in which there are caps on consumption.
2. In our model, agents evaluate their utility considering all the consequences involved in their consumption plan and in other's.
3. We have shown that under very weak conditions on the fundamentals of the economy, equilibrium exists and, given the status quo, the equilibrium allocation is Pareto efficient.

4. As a consequence of Walras Law, an effective cap on a commodity implies that the equilibrium price of that commodity is zero and then the price of the consumption right becomes the relevant price.

   An agent that is aware of the effectiveness of the cap on a raw material and holds in her endowment an important amount of it, could have incentives withholding it.

   Moreover, given the relevance of the corresponding allowance price, the distribution of rights among agents must be important in order to avoid withholding.

5. We are not considering the political welfare aspects derived from the distribution of the allowances among the agents. This will be analyzed in a future study.