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Desperately Seeking θ's: Estimating the Distribution of Consumers Under Increasing Block Rates*

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Abstract
This paper shows that increasing block rate pricing schedules usually applied by water utilities can reduce the efficiency and equity levels. To do this, we first present a two step method to estimate the demand and to recover the distribution of consumer tastes when increasing block rate pricing is used. We show that in this case the tariff induces a pooling equilibrium and customers with different taste parameters will be observed to choose the same consumption level. Second, we show that a two-part tariff that neither reduces the revenue for the firm nor increases the aggregate level of water consumption increases the welfare and equity levels in relation to an increasing block rates schedule.

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1. Introduction

Water utilities regulators frequently use increasing-block rate pricing. Indeed, OECD reports that this pricing rule is used by 31% of the utilities in the U.S., 27% in Australia, 57% in Japan, 74% in Mexico, 90% in Spain, and 100% in Belgium, Greece, Italy, Korea and Portugal.[1] Moreover, the increasing-block pricing system includes a fixed charge in most of these countries. For example, 100% of the tariffs in Belgium, Italy, Australia, Korea, Turkey and Spain have this characteristic (OECD 1999).

In other countries, like Germany, the government advices utilities to utilize increasing block rates (see OECD, 1999). It is well-known that the main fundamentals for using this pricing schedule are twofold (''[regulators] use the price policy with the dual objective of improving equity among customers and achieving some reduction in total water use'', Agthe and Billings 1987). Firstly, in areas subject to overdraft of an aquifer, increasing block rate price schedules are imposed to reduce usage and achieve intergenerational equity. High rates for the largest volume users, who are supposedly the most affluent, are imposed not only to restrict water use but also to reduce the amount of capital investment needed to provide peak load service to large volume users. Secondly, an increasing block rate pricing structure might be utilized to improve interpersonal equity. If high income households use more water than lower income households, and water is subject to diminishing marginal utility, the marginal utility derived from the last unit of water consumed is smaller for high income households than for low income households. Therefore, to achieve equity, above average rates on higher income users should be imposed while offering below average rates to low income users.[2]

In this paper we show that the increasing-block rate pricing induces pooling equilibria and that the estimation of the demand will be biased if we use the distribution of observed consumption as a proxy for the distribution of consumers' preferences. We also show that there can exist other pricing schedules that allow for the achievement of larger efficiency and equity levels. In particular, we find that a two-part tariff that neither increases the aggregate level of water consumption nor reduces current revenues of the firm increases the total welfare. Furthermore, this new tariff improves the equity because it reduces the payments and average prices of the consumers with the lowest consumption levels. Our results suggest that the use of increasing-block rates with a fixed charge hurt the low income users and worsen equity and efficiency.

To recover the distribution of consumers we need to know which levels of consumption are demanded by more than one type of customers, and how many customers demand the

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1 OECD (1999). *The Price of the Water*. OECD, Paris.

2 In this sense an increasing block rate pricing schedule is similar to a form of ''Ramsey pricing'' in which different units of the same commodity are interpreted as different products. At each point on the outlay schedule, the percentage mark-up between marginal price and marginal cost is inversely proportional to the elasticity of demand for an increment of consumption at that point.

Public water providers may also use increasing block rate when there exist externalities arising from certain uses of water. For example, when a public health externality results from everyone having access to potable water, a low marginal price for an amount of water per month required for adequate sanitation, accompanied with relatively high marginal prices for all additional water use, could be optimal.

same quantity. In this sense, we can say that these individuals are hidden and we need ''to desperately seek them''.[3] Thus, our primary contribution is to present a two-step method to estimate the demand when increasing block rate pricing in combination of a minimum charge is used. First, we estimate the demand function induced by the current tariff. In particular, we estimate the demand relation for each block as a function where the price variables are the marginal price of that block and the marginal prices of the adjacent blocks. This allows us to write the observed distribution of consumption as a function of the theoretical distribution of consumers induced by the current tariff. Second, we use the observed frequencies of consumption to estimate the distribution parameters. This allows us to obtain an estimation of the number of consumers induced to demand the same amount of water, the hidden consumers. To illustrate this technique, we use data from a municipal water utility in Spain.

Once the true distribution of consumer types has been estimated, we look at the equity and welfare effects of changing the tariff. In particular, we introduce a two-part tariff which increases equity and overall efficiency in relation to an increasing block rate pricing schedule. Thus, we show that consumers who were consuming a quantity lower than $20\,m^3$ or higher than $35\,m^3$ are better off with the new tariff. Concerning the consumers who are located between 20 and $35\,m^3$, 56% improves their welfare and 54% worsen. We also contend that total welfare is higher with the new tariff. These welfare gains only come from the reallocation of the actual level of aggregate consumption and not from increased aggregate consumption and redistribution across consumers. This seems to be an important point providing water is a scarce resource. Moreover, this fact allows us to better evaluate the ''equity'' of increasing block rates.

The analysis of optimal screening of agents with unknown characteristics and bunching issues in nonlinear pricing, optimal taxation, regulation of public utilities, and so on, has been the subject of a great amount of theoretical literature over the last years. Rochet and Chonè (1998) present a brief survey of this literature. However, we do not know of any prior empirical study that deals with the bunching issue. Our paper is similar to Mitchell (1978) in the sense that we also use the frequency distribution of consumption to retrieve the distribution of consumer tastes, and we simply use a different assumption about the distribution of preferences. In particular, we assume that consumer preferences are determined by unobserved individual characteristics drawn from a Weibull distribution. However, to solve the problem of pooling at particular consumption levels, we explicitly include in the estimation procedure the relation between consumption and taste parameter induced by the tariff. In this sense, our work is close to Ivaldi and Martimort (1994) in the way in which fundamental parameters pertaining to demand and supply sides are obtained

3 We assume that customers present similar ''production functions'' and that all differences in consumption come from different tastes for the good. There exists the possibility that customers may be in different positions with respect to the good. That is, consumption may differ because of different ''production functions'' or ''cost functions'', not just because of different tastes for the good. In fact, if substitutability for alternative goods differs then we might observe wide differences in consumption without it being true that there is wide variation in the intrinsic utility derived from the good. Petrin (1998) has shown that precisely this error probably led to overestimates of the value of minivans to automobile consumers on the order of a factor of four.

by estimating a structural model. Finally, the efficiency analysis is closely related to the literature that evaluates the welfare effects from reforming pricing schedules in utilities by using aggregate level data (e.g., Mitchell 1978; Brown and Sibley 1986; Dimopoulos 1981; Renzetti 1992; Gabel and Kennet 1993; Crandall and Waverman 1995).

This paper is organized as follows. Section 2 presents the problem. Section 3 proposes a simple model. We show how a simple tariff with a minimum charge and only a marginal price induces pooling equilibria. In section 4 we estimate the parameters of the demand function and the true distribution of the consumers types. In section 5 we analyze the effects on welfare, equity and inequality of introducing a two-part tariff. Finally, in section 6 the conclusion is provided.

2. The Problem

The water tariff in use in the city of Vigo, Spain, from January 1992 to April 1993, displays the following structure. It has a minimum charge A, which includes a free allowance of q_0 m^3. When the level of consumption exceeds q_0, the consumer must pay p_i for each unit. The price p_i depends on the level of consumption:

$$p_i = \begin{cases} p_1 & \text{if } q_0 < q \leq q_1 \\ p_2 & \text{if } q_1 < q \leq q_2 \\ p_3 & \text{if } q_2 < q, \end{cases}$$

where $p_3 > p_2 > p_1 > 0$, and $q_2 > q_1 > q_0$. That is to say, the marginal price is increasing in consumption.

Figure 1 shows the frequency distribution of residential consumptions of water. This distribution is asymmetric and displays a high concentration of consumption at about 30 m^3, exactly the volume of service associated with the minimum charge (consumption minima).

This paper shows that the high concentration at 30 m^3 means that consumers with different tastes for the good are choosing the same consumption level. Thus, when estimating the demand, we cannot use the distribution of observed consumption as a proxy for the distribution of consumers, because there are different types of customers consuming the same level of consumption. Hence, to recover the true distribution of consumer types we need to know how many customers demand the same quantity.

In the next section we propose a simple model that explains in detail the stylized fact of consumers' concentration in the minimum of consumption.

3. The Model

Following Mitchell (1978) we assume that all the differences between consumers can be represented by a taste parameter. More specifically, we assume that all consumers have the same reservation price and the differences among them arise from the different level of satiation. Then, the consumers' preferences can be represented as follows:

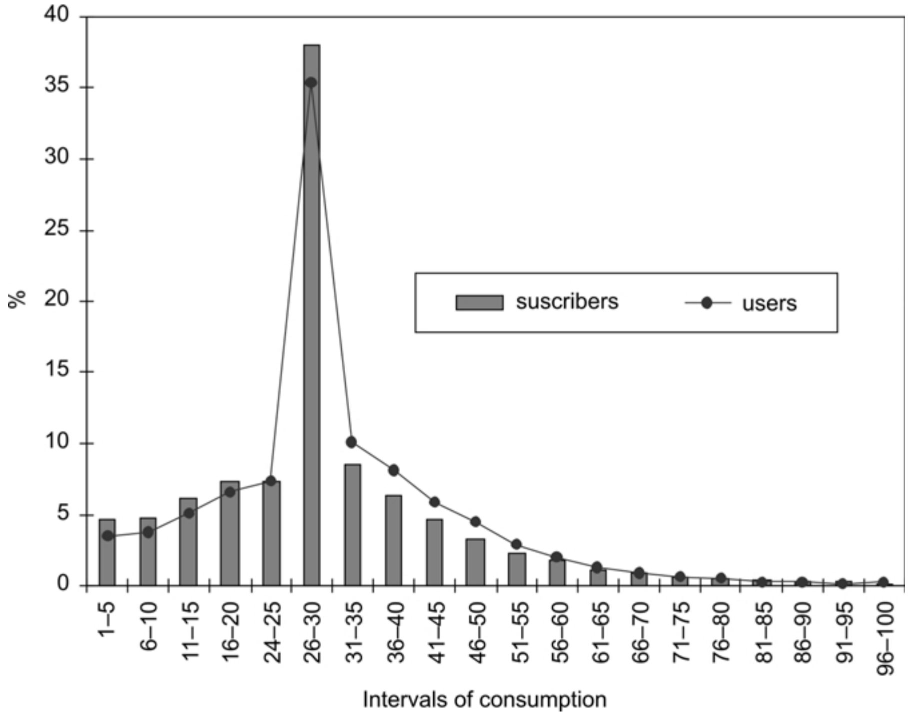

Figure 1. Frequencies of consumers.

$$V_\theta(q,T) = \begin{cases} U(q,\theta) - T & \text{if } q > 0 \\ 0 & \text{if } q = 0, \end{cases}$$

where q is the level of consumption, T is the total payment and θ is a taste parameter that varies among consumers (we will call it the consumer's type) with support $[\underline{\theta}, \overline{\theta}]$. We will assume a quadratic utility function in consumption

$$U(q,\theta) = \alpha q - \frac{1}{2\theta} q^2, \qquad (1)$$

where α, the reservation price, is equal for all the consumers.[4] Note that consumers are different because they have different levels of satiation, $\alpha\theta$.[5]

In this section we analyze a simplified model where the tariff displays only a marginal

4 We assume that the consumers have a separable utility function for water and other goods. This implies that there is no income effect on the water demanded (see Mitchel 1978). On the other hand, the possible ''virtual income'' effect created by the nonlinear budget constraint is negligible for a good such as the water service, which represents a small fraction of a consumer's budget.

5 Although in many applications the satiation property is unnecessarily restrictive, in this example it is a virtue, since the demand for water must be finite at a zero price.

price. Therefore, this model will be generalized. If the public utility charges for the good a tariff with a fixed fee A which includes a free allowance of q_0 units and p for each unit if the level of consumption exceeds q_0, the total payment $T(q)$ is

$$T(q) = \begin{cases} A & \text{if } q \leq q_0 \\ pq & \text{if } q > q_0. \end{cases}$$

There is another way to interpret the tariff. If the level of consumption exceeds q_0, the consumers pay A plus an additional payment of $p - p_0$ for the consumption minima q_0 and p for the $q - q_0$ remaining units. Formally, we can rewrite the total payment, $T(q)$, as

$$T(q) = \begin{cases} A & \text{if } q \leq q_0 \\ A + (p - p_0)q_0 + p(q - q_0) & \text{if } q > q_0, \end{cases}$$

where p_0 is the implicit ''price'' of the free allowance q_0 units, defined as $p_0 = A/q_0$.

To obtain the demands we must solve the consumer problem, $\max_q\{U(q, \theta) - T(q)\}$. In order to do this, it is useful to define the consumer who will not be consuming beyond the limit of the lower interval of prices; formally:

$$\theta_0 = \frac{q_0}{\alpha} \tag{2}$$

Note that θ_0 is independent of the prices because the consumer who demands q_0 pays a fixed fee.

The existence of a consumption minima, q_0, induces all the consumers with type $\theta \leq \theta_0$ to consume their level of satiation $\alpha\theta$. We might expect that consumers with $\theta > \theta_0$ will consume a higher amount of the good. Nevertheless, some of them may prefer to consume q_0 providing that, if they consume more, they must pay more for the first q_0 units.

Proposition 1: *All the consumers with types $\theta \in [\theta_0, \theta_0^*]$, with $\theta_0 = q_0/\alpha$ and*

$$\theta_0^* = \frac{(\alpha - p_0) + \sqrt{2(p - p_0)(\alpha - p_0 - \frac{\Delta p}{2})}}{(\alpha - p)^2} q_0,$$

exactly consume q_0, the consumption minima.

Proof: A consumer with $\theta > \theta_0$ demands $q^* = \theta(\alpha - p)$ if and only if $q^* = \max_q\{U(q, \theta) - T(q)\}$, for all q. In particular it must be true that

$$U(q^*, \theta) - pq^* \geq U(q_0, \theta) - A.$$

Let θ_0^* be the consumer who is indifferent between consuming q_0 or his best choice if he decides to demand $q > q_0$. Formally, θ_0^* satisfies:

$$U(\theta_0^*(\alpha - p), \theta_0^*) - p\theta_0^*(\alpha - p) = U(q_0, \theta_0^*) - p_0 q_0.$$

Some algebra allows us to obtain

$$\theta_0^* = \frac{(\alpha - p_0) \pm \sqrt{2\Delta p\left(\alpha - p_0 - \frac{\Delta p}{2}\right)}}{(\alpha - p)^2} q_0,$$

where $\Delta p = p - p_0$.[6] Now, it is easy to show that the welfare gain of a type θ when he consumes a level of consumption higher than q_0 is

$$\begin{aligned}\Delta W(\theta) &= U(q^*, \theta) - U(q_0, \theta) - (p - p_0)q_0 - p(q^* - q_0) \\ &= \frac{1}{2}\left[\theta(\alpha - p)^2 + \frac{q_0^2}{\theta}\right] - (\alpha - p)q_0.\end{aligned}$$

Then if $\theta \geq \theta_0^*$, $q(\theta) = q^*$, and if $\theta < \theta_0^*$, $q(\theta) = q_0$, given that

$$\frac{d\Delta W(\theta)}{d\theta} = \frac{1}{2}\left[(\alpha - p)^2 - \frac{q_0^2}{\theta^2}\right] > 0,$$

since $q(\theta) = \theta(\alpha - p)$ must be higher than q_0, because otherwise it is trivial that the consumer chooses $q = q_0$ (he consumes more and pays less). ■

Corollary: *All the consumers with type* $\theta \in [\theta_0, \theta_0^*]$ *consume exactly the consumption minima, thus the model exhibits pooling equilibrium.*

So the model predicts a high concentration of consumers in q_0. We can write the individual consumer demands as follows:

$$q(\theta) = \begin{cases} \theta\alpha & \text{if } \underline{\theta} \leq \theta < \theta_0 \\ q_0 & \text{if } \theta_0 \leq \theta \leq \theta_0^* \\ \theta(\alpha - p) & \text{if } \theta_0^* < \theta \leq \overline{\theta}. \end{cases} \tag{3}$$

In general, if we consider a tariff with k marginal prices, when the price of the good changes from p_k to p_{k+1}, there are some consumers of type $\theta > \theta_k$, who prefer to consume q_k because in any other case they would pay more for the first q_k units and the increase in the bill is higher than the additional welfare obtained by increasing the consumption of the

6 We always take the solution with the positive sign, because it is easy to verify that $\theta_k^{*(-)} \leq \theta_k \leq \theta_k^{*(+)}$ for $k = 1, 2$ and if α is big enough (specifically, greater than 226.12) it is also true for $k = 0(\theta_k^{*(-)}$ and $\theta_k^{*(+)}$ are the solutions of (3) with negative and positive signs of the square root). It will be shown later that α must be greater than 226.12.

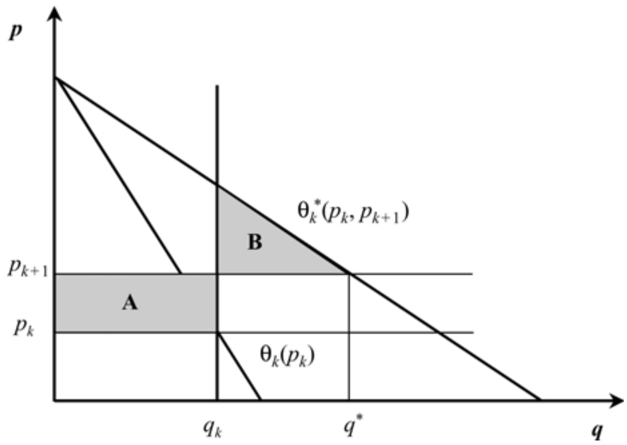

Figure 2. Consumers type θ in $[\theta_\kappa, \theta_\kappa^*]$ prefer consume q_k.

good. This can be seen in figure 2, which presents the demand functions of two consumer types, θ_k and θ_k^*. The consumer θ_k consumes q_k at price p_k, the maximum level of consumption allowed at that price. That is to say, $\theta_k = q_k/(\alpha - p_k)$. In general, all consumers with type parameter $\theta > \theta_k$ will consume a higher level of consumption because it increases their surplus in area B. However, by consuming a level of consumption higher than q_k, one must pay a higher marginal price p_{k+1}, and the bill for the first q_k units is increased in area A. This means that those consumers of type $\theta > \theta_k$ with area A bigger than area B consume the same level of consumption q_k. For the consumer type θ_k^*, both areas are equal and he is indifferent between q_k and q_k^*.

4. Model Estimation

The last section showed that when the tariff displays increasing block rates with a minimum charge, different types of consumers are induced by the current tariff to consume the same amount of the good. This implies that we cannot use the observed consumption distribution as a proxy for the distribution of types. Nevertheless, it is possible to specify the consumption distribution as a function of the actual distribution of types.

We now present the above model in a way that allows empirical implementation. To obtain the demands with three marginal prices $p_1 < p_2 < p_3$, it is useful to define the consumers who are sure to be consuming in the limit of each interval of prices; formally:

$$\theta_0 = \frac{q_0}{\alpha}, \quad \theta_k(p_k) = \frac{q_k}{\alpha - p_k}, \quad k = 1, 2. \tag{4}$$

Let θ_k^* be the consumer who is indifferent between consuming q_k or his best choice if he decides to demand $q > q_k$, $q_k^* = \theta_k^*(\alpha - p_{k+1}) > q_k$. Formally θ_k^* satisfies:

$$U(q_k, \theta_k^*) - p_k q_k = U(\theta_k^*(\alpha - p_{k+1}), \theta_k^*) - p_{k+1}\theta_k^*(\alpha - p_{k+1}).$$

So we obtain

$$\theta_k^* = \frac{(\alpha - p_k) + \sqrt{2\Delta p_k\left(\alpha - p_k - \frac{\Delta p_k}{2}\right)}}{(\alpha - p_{k+1})^2} q_k, \quad k = 0, 1, 2,$$

where $\Delta p_k = p_{k+1} - p_k$. Then all the consumers with types $\theta \in [\theta_k, \theta_k^*]$ prefer to consume q_k. Then we can write the individual consumer demands as follows:

$$q(\theta) = \begin{cases} \theta\alpha & \text{if } \theta < \theta_0 \\ q_0 & \text{if } \theta_0 \leq \theta \leq \theta_0^* \\ \theta(\alpha - p_1) & \text{if } \theta_0^* < \theta < \theta_1 \\ q_1 & \text{if } \theta_1 \leq \theta \leq \theta_1^* \\ \theta(\alpha - p_2) & \text{if } \theta_1^* < \theta < \theta_2 \\ q_2 & \text{if } \theta_2 \leq \theta \leq \theta_2^* \\ \theta(\alpha - p_3) & \text{if } \theta_2^* < \theta. \end{cases} \tag{5}$$

Given the relation (5) between consumption and type, the probability distribution of the variable q in the space of consumptions depends on the types distribution and the parameters α, θ_0^*, θ_1^* and θ_2^*. Let F_q be the distribution function of q, defined as

$$F_q(q) = \begin{cases} F_\theta(q/\alpha) & \text{if } q < q_0 \\ F_\theta(\theta_0^*) & \text{if } q_0 \leq q < \theta_0^*(\alpha - p_1) \\ F_\theta(q/(\alpha - p_1)) & \text{if } \theta_0^*(\alpha - p_1) \leq q < q_1 \\ F_\theta(\theta_1^*) & \text{if } q_1 \leq q < \theta_1^*(\alpha - p_2) \\ F_\theta(q/(\alpha - p_2)) & \text{if } \theta_1^*(\alpha - p_2) \leq q < q_2 \\ F_\theta(\theta_2^*) & \text{if } q_2 \leq q < \theta_2^*(\alpha - p_3) \\ F_\theta(q/(\alpha - p_3)) & \text{if } \theta_1^*(\alpha - p_2) \leq q, \end{cases} \tag{6}$$

where F_θ is the distribution function of θ.

We use data from a municipal water utility (Vigo, Spain) to show that when there exist tariffs with a consumption minima, we need to recover the underlying fundamentals of the demand. In this section, we first present the way in which the above model fits the data. Second, the econometric methodology to estimate the model and the estimation results are also outlined.

The reservation price and the number of hidden consumers are calculated in two steps. First, we estimate the reservation price α from a demand system. Given α (or its estimation), F_θ can be estimated (jointly with $\theta_k^*, k = 0, 1, 2$) from the observed consumption distribution F_q. Then, the hidden consumers in each interval of consumption can be quantified.

4.1. Estimating the Reservation Price α

Let us start aggregating the individual demands for each interval of consumption. Thus, we can write the total demand in each interval (Q_k, $k = 0, 1, 2, 3$) as a function of the current prices for this interval and the prices of the adjacent intervals. Formally,

$$\begin{aligned}
Q_0(p_0, p_1) &= N\left(\int_{\underline{\theta}}^{\theta_0} \alpha\theta f(\theta)d\theta + \int_{\theta_0}^{\theta_0^*(p_0,p_1)} q_0 f(\theta)d\theta\right),\\
Q_1(p_0, p_1, p_2) &= N\left(\int_{\theta_0^*(p_0,p_1)}^{\theta_1} \theta(\alpha - p_1) f(\theta)d\theta + \int_{\theta_1}^{\theta_1^*(p_1,p_2)} q_1 f(\theta)d\theta\right),\\
Q_2(p_1, p_2, p_3) &= N\left(\int_{\theta_1^*(p_1,p_2)}^{\theta_2} \theta(\alpha - p_2) f(\theta)d\theta + \int_{\theta_2}^{\theta_2^*(p_2,p_3)} q_2 f(\theta)d\theta\right),\\
Q_3(p_2, p_3) &= N\int_{\theta_2^*(p_2,p_3)}^{\overline{\theta}} \theta(\alpha - p_3) f(\theta)d\theta,
\end{aligned} \tag{7}$$

where N is the size of the whole population. Given that the aggregate demands by interval are multiplicatively separable, it is useful to normalize Q_k by the total number of consumers N. So, we define the normalized demands $D_k(\cdot)$ as $D_k = Q_k/N, k = 0, 1, 2, 3$.

The system allows us to obtain α from $\partial D_0/\partial p_0$ and $\partial D_1/\partial p_0$, providing that[7]

$$\frac{\partial D_1}{\partial p_0} = -\theta_0^*(\alpha - p_1) f(\theta_0^*)\frac{d\theta_0^*}{dp_0},$$

and

$$\frac{\partial D_0}{\partial p_0} = q_0 \frac{d\theta_0^*}{dp_0} f(\theta_0^*),$$

it follows that

$$-\frac{\partial D_1/\partial p_0}{\partial D_0/\partial p_0} = \frac{\theta_0^*(\alpha - p_1)}{q_0}.$$

Remember that (2) implies that θ_0^* is

$$\theta_0^* = \frac{(\alpha - p_0) + \sqrt{2\Delta p_0\left(\alpha - p_0 - \frac{\Delta p_0}{2}\right)}}{(\alpha - p_1)^2} q_0.$$

7 The partial derivatives are calculated in Appendix A.1.

Then, α can be calculated from the expression

$$\Phi(\alpha) = \frac{\partial D_1/\partial p_0}{\partial D_0/\partial p_0} + \frac{\alpha - p_0 + \sqrt{2\Delta p_0\left(\alpha - p_0 - \frac{\Delta p_0}{2}\right)}}{(\alpha - p_1)} = 0, \tag{8}$$

where $\Delta p_0 = p_1 - p_0$.

The water supplier in Vigo provides both water and sewerage service. Seven areas of supply are distinguished, according to their amounts of sewage (see the map of the city in the Appendix). As can be observed in the Appendix, the closer to the center the zone is, the higher the proportion of collected sewage. Hence, for instance, zone III, located in the city center, displays the highest average collected sewage, while zone VI, situated in a rural area far from the city center, displays the lowest average collected sewage.

In order to estimate the partial derivatives of the normalized demands in each interval of consumption, we use per capita levels of consumption in that interval, $\overline{q}_k$, for several zones in the city, $\bar{x}_k^t$. We can compute the normalized demands as

$$d_k = \bar{x}_k \frac{N_k}{N},$$

where N_k is the number of consumers in each interval.

On the other hand, as the price tariff includes both a supply charge and a sewage charge, the effective average price of water will be different for each interval and zone. In particular, we have calculated the average price for the interval $k = 0, 1, 2, 3$, and the zone $t = \mathrm{I}, \ldots, \mathrm{VII}$, as

$$p_k^t = p_k^t(\text{supply}) + \gamma_k^t p_k^t(\text{drain}),$$

the sum of the supply charge and sewage charge weighted by the proportion of collected sewage in that interval and zone, γ_k^t (the zones in use and its coefficients are listed in the Appendix).

In this way we build a sample from which we can estimate a linear approximation to the system (7) where the coefficients are the partial derivatives of D_i with respect to the prices. The linear system of equations is

$$\begin{bmatrix} d_0^t \\ d_1^t \\ d_2^t \\ d_3^t \end{bmatrix} = \begin{bmatrix} \beta_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \beta_{00} & \beta_{01} & 0 & 0 \\ \beta_{10} & \beta_{11} & \beta_{12} & 0 \\ 0 & \beta_{21} & \beta_{22} & \beta_{23} \\ 0 & 0 & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} p_0^t \\ p_1^t \\ p_2^t \\ p_3^t \end{bmatrix} + \begin{bmatrix} \varepsilon_0^t \\ \varepsilon_1^t \\ \varepsilon_2^t \\ \varepsilon_3^t \end{bmatrix}, \tag{9}$$

where β_{ij} is the partial derivative of D_i with respect to p_j. Note that the water demand has an independent term only in the first interval of consumption (β_0). From the estimation of the system, we can estimate α replacing $\partial D_1/\partial p_0/\partial D_0/\partial p_0$ by β_{10}/β_{00} in equation (8).

The coefficients in (9) are subject to several restrictions required by the model and by the observed data. The restrictions are as follows:

- **Symmetry.** Appendix proves that $\beta_{01} = \beta_{10}$, $\beta_{12} = \beta_{21}$ and $\beta_{23} = \beta_{32}$.
- **Sign of the coefficients.** It is also proved in the Appendix that demands in each interval are decreasing functions of their own price and increasing functions of the neighboring prices. So,

$$\beta_{ij} < 0 \text{ if } i = j,$$
$$\beta_{ij} > 0 \text{ if } i \neq j.$$

- The saturated agents' consumption is less than or equal to that observed in the first interval of consumption, so

$$\beta_0 \leq d_0(p_0, p_1).$$

- $H = -(\beta_{01}/\beta_{00}) > 1$ because $\Phi(\alpha) = 0$ implies

$$H = \frac{\alpha - p_0 + \sqrt{2\Delta p\left(\alpha - p_0 - \frac{\Delta p}{2}\right)}}{(\alpha - p_1)} > \frac{\alpha - p_0}{\alpha - p_1} > 1.$$

- ***Value of α.*** The estimation of α has to be compatible with the consumptions under the present rate. So, we define the consumer with the lowest valuation of the good $(\underline{\theta})$ as the one who buys a cubic meter of water. His gross profit is $\alpha/2$. Now he is paying the minimum bill (T = 2280 pesetas), that is,

$$\alpha \geq 4560.$$

  If such a restriction holds, then only the positive sign in the squared root involved in the definition of θ_0^* is compatible with the fact that $\theta_0 < \theta_0^*$.
- **Restrictions on the proportion of hidden consumers.** The estimation of the proportion of hidden consumers has to be lower than the observed proportion of people consuming just the quantities that separate two intervals (see the users' frequency in section 6.2.4; the proportion of users consuming between 196 and 200 is not reflected there because consumptions over 100 were aggregated; its true value is 0.0002). So,

$$0 \leq \int_{\theta_0}^{\theta_0^*} f(\theta)d\theta = \frac{23,67 - \beta_0}{30} \leq 0.3536,$$

$$0 \leq \int_{\theta_1}^{\theta_1^*} f(\theta)d\theta = \frac{43.34 + K_1(\alpha - 70.5)}{70} \leq 0.0081,$$

$$0 \leq \int_{\theta_2}^{\theta_2^*} f(\theta)d\theta = \frac{97.02 + K_2(\alpha - 83.5)}{200} \leq 0.0002,$$

where K_i (defined in Appendix) are functions of β.

- From the definition of K_3 and the expression of the aggregate demand in the third interval, it follows that

$$\frac{d_3(p_2, p_3)}{\alpha - p_3} + K_3 = 0.$$

Taking into account the symmetry of the restrictions, the linear system to be estimated can be written as

$$X = P\beta + \varepsilon, \ \varepsilon \sim (0, \Sigma),$$

where X and ε have dimension 4, P is a 4×8 matrix and β is of dimension 8. The coefficient vector is $\beta = (\beta_0, \beta_{00}, \beta_{01}, \beta_{11}, \beta_{12}, \beta_{22}, \beta_{23}, \beta_{33})^t$. Appendix includes the available data $(X_i, P_i), \ i = 1, \ldots, n$.

We define

$$\mathscr{X} = (X_1^t, \ldots, X_n^t)^t,$$
$$\mathscr{P} = (P_1^t, \ldots, P_n^t)^t,$$
$$\mathscr{E} = (\varepsilon_1^t, \ldots, \varepsilon_n^t)^t,$$
$$\mathscr{S} = I_n \otimes \Sigma,$$

where $\otimes$ denotes the Kroneker product. So, we can transform the original linear system into a regression model with the variance of the residuals different from the identity matrix:

$$\mathscr{X} = \mathscr{P}\beta + \mathscr{E}, \ \mathscr{E} \sim (0, \mathscr{S}).$$

We propose an estimator of β similar to the generalized least squares estimator (GLS) based on a previous estimation of the matrix $\mathscr{S}$. First, we take the vector minimizing the sum of the squared norms of the residual, subject to the restrictions previously enumerated, as an estimator of β. Then, we compute the residuals from that estimator and Σ is estimated by the sample covariance matrix of these residuals. Finally, a second estimation of β is carried out in the same way as the GLS estimator would be applied (i.e., using the estimation of $\mathscr{S}$ based on those of Σ instead of the unknown variance matrix). The restrictions of the coefficients also have to be taken into account in this second step. The algorithm used in the minimization phases is based on penalty functions. The final estimation of β is

$$\hat{\beta} = (7.12, -1.62, 1.72, -89.26, 21.86, -7.56, 0.000011, -0.000012)^t.$$

From these values we arrive to an estimation of α equal to 4560, the lower bound for this parameter. An interesting spin-off of the system estimation is the evaluation of values θ_k^*, because equation (3) expresses the values of θ_k^* as functions of α. Then, we obtain the estimations

$$\theta_0^* = 0.71 \times 10^{-2}, \quad \theta_1^* = 0.17 \times 10^{-1}, \quad \text{and} \quad \theta_2^* = 0.50 \times 10^{-1}.$$

4.2. Estimating the Distribution of Types

The available sample information refers to consumptions (see the Appendix). The consumption space is divided into 21 intervals, each of them 5 m^3 wide (except the last one). Only aggregate information for each interval is available. The column with the percentage of users is used as the frequency of variable q.[8] Let $f_i, \ i = 1, \ldots, 21$, be the observed relative frequencies of q.

In order to make the theoretical continuous distribution F_q compatible with the sample information, essentially discrete, we compute the probability assigned by F_q to each of the 21 intervals of consumption for which we know the observed frequency. These theoretical probabilities $\pi_i, \ i = 1, \ldots, 21$, are

$$\pi_i = \begin{cases} F_q(5i) - F_q(5i-5) & \text{if } i \leq 20 \\ 1 - F_q(5i-5) & \text{if } i = 21. \end{cases} \tag{10}$$

The demand $q(\theta)$ defined in (5) implies that π_i depends on the distribution of types F_θ. We impose continuity, that is $q_k = \theta_k^*(\alpha - p_{k+1}), \ k = 0, 1, 2$, to derive from (5) the following transformation:

$$q(\theta) = \begin{cases} \theta\alpha & \text{if } \theta < \theta_0 \\ q_0 & \text{if } \theta_0 \leq \theta \leq \theta_0^* \\ q_0 + (\theta - \theta_0^*)(\alpha - p_1) & \text{if } \theta_0^* < \theta < \theta_1 \\ q_1 & \text{if } \theta_1 \leq \theta \leq \theta_1^* \\ q_1 + (\theta - \theta_1^*)(\alpha - p_2) & \text{if } \theta_1^* < \theta < \theta_2 \\ q_2 & \text{if } \theta_2 \leq \theta \leq \theta_2^* \\ q_2 + (\theta - \theta_2^*)(\alpha - p_3) & \text{if } \theta_2^* < \theta. \end{cases} \tag{11}$$

8 Subscribes represents the number of meters and users represents the number of consumers in each meter.

To relate π_i and f_i we need to propose a parameterization for F_θ. Given that observed consumptions are very concentrated in the lower values and are very asymmetric, a sensible model for F_θ is the Weibull distribution with scale parameter μ and shape parameter ρ. The density function of such a distribution is

$$f_\theta(\theta;\mu,\rho) = \frac{\mu}{\rho^\mu}\theta^{\mu-1}e^{-(\theta/\rho)^\mu},$$

and its distribution function is

$$F_\theta(\theta;\mu,\rho) = 1 - e^{-(\theta/\rho)^\mu}.$$

The definition of π_i given in equation (10) and the relation between F_θ and F_q established in equation (6) leads to the following closed expression for π_i as a function of some unknown parameters: $\pi_i = \pi_i(\mu,\rho,\theta_0^*,\theta_1^*),\ i = 1,\ldots,21$:

$$\pi_i = \begin{cases} \exp\left\{-\left(\frac{5(i-1)}{\alpha\rho}\right)^\mu\right\} - \exp\left\{-\left(\frac{5i}{\alpha\rho}\right)^\mu\right\} & \text{if } i \le 5 \\ \exp\left\{-\left(\frac{5(i-1)}{\alpha\rho}\right)^\mu\right\} - \exp\left\{-\left(\frac{5(i-6)}{(\alpha-p_1)\rho}+\frac{\theta_0^*}{\rho}\right)^\mu\right\} & \text{if } i = 6 \\ \exp\left\{-\left(\frac{5(i-7)}{(\alpha-p_1)\rho}+\frac{\theta_0^*}{\rho}\right)^\mu\right\} - \exp\left\{-\left(\frac{5(i-6)}{(\alpha-p_1)\rho}+\frac{\theta_0^*}{\rho}\right)^\mu\right\} & \text{if } 7 \le i \le 13 \\ \exp\left\{-\left(\frac{5(i-7)}{(\alpha-p_1)\rho}+\frac{\theta_0^*}{\rho}\right)^\mu\right\} - \exp\left\{-\left(\frac{5(i-14)}{(\alpha-p_2)\rho}+\frac{\theta_1^*}{\rho}\right)^\mu\right\} & \text{if } i = 14 \\ \exp\left\{-\left(\frac{5(i-15)}{(\alpha-p_2)\rho}+\frac{\theta_1^*}{\rho}\right)^\mu\right\} - \exp\left\{-\left(\frac{5(i-14)}{(\alpha-p_2)\rho}+\frac{\theta_1^*}{\rho}\right)^\mu\right\} & \text{if } 15 \le i \le 20 \\ \exp\left\{-\left(\frac{5(i-15)}{(\alpha-p_2)\rho}+\frac{\theta_1^*}{\rho}\right)^\mu\right\} & \text{if } i = 21. \end{cases} \tag{12}$$

The value of α, which we obtained in the previous section ($\alpha = 4560$), is part of the definition of π_i because of equation (6). Note also that π_i does not depend on θ_2^* because the consumption related to that consumer is $q = 200$ and it belongs to the last interval of consumptions $(q > 100)$. After the division of the consumptions into 21 intervals, the consumption space can be understood as a discrete space of probability with a support set of 21 points. There we have defined a parametric multinomial probability law given by the mass function $(\pi_1,\ldots,\pi_{21})$, depending on μ, ρ, θ_0^* and θ_1^* as equation (12) shows. We have also observed frequencies for these 21 points: $(f_1,\ldots,f_{21})$.

There exist several econometric tools for the estimation of the unknown parameters from the observed frequencies. A good reference for this kind of estimation is Read and Cressie (1988). They deal with the problem of estimation in multinomial distributions where the k cells probabilities (in our case, $\pi_i, i = 1,\ldots,21$) are given as a function of a s-dimensional parameter, say ϑ, (in our case, $k = 21$, $s = 4$ and equation (12) determines the functional dependence between π_i and $\vartheta = (\mu,\rho,\theta_0^*,\theta_1^*)$). They study the family of minimum power-divergence estimators of ϑ from the observed frequencies f_i (see Appendix 5 of Read and Cressie 1988), where both the maximum likelihood estimator and the minimum χ^2 estimators of ϑ are included. They prove that under some regularity conditions (fulfilled in our case) all the estimators in that family are best asymptotically normal (BAN) estimators, that is to say, all of them are as efficient as the maximum likelihood estimator (see Theorem A5.1 in Read and Cressie, 1988). The asymptotic distribution of those estimators is given by Corollary A5.1 in Read and Cressie (1988), which can be expressed as follows:

$$\sqrt{n}(\hat{\vartheta} - \vartheta^0) \to_w N_s(0, (A'A)^{-1}),$$

where ϑ^0 is the unknown true value of the parameter ϑ, $\hat{\vartheta}$ is any of the minimum power-divergence estimators of ϑ, n is the sample size, and A is the $k \times s$ matrix with (i,j)th element equal to

$$(\pi_i^0)^{-1/2} \frac{\partial \pi_i(\vartheta^0)}{\partial \theta_j},$$

having defined $\pi_i^0 = \pi(\vartheta^0)$ and being k the total number of cells in the multinomial distribution definition.

Based on this result, in this study we use the minimum χ^2 estimation procedure. It consists of minimizing the χ^2 statistic used in the goodness of fit test of the observed frequencies $(f_1, \ldots, f_{21})$ to the theoretical probabilities $(\pi_1, \ldots, \pi_{21})$:

$$T(\mu, \rho, \theta_0^*, \theta_1^*) = \sum_{i=1}^{21} \frac{(nf_i - n\pi_i)^2}{n\pi_i} = n \sum_{i=1}^{21} \frac{(f_i - \pi_i)^2}{\pi_i}.$$

So, we minimize the function

$$\Psi(\mu, \rho, \theta_0^*, \theta_1^*) = \sum_{i=1}^{21} \frac{(f_i - \pi_i(\mu, \rho, \theta_0^*, \theta_1^*))^2}{\pi_i(\mu, \rho, \theta_0^*, \theta_1^*)}$$

in the unknown parameters μ, ρ, θ_0^* and θ_1^*.

Table 1 presents the results of the estimation and table 2 shows the correlation matrix of the parameters estimators. A comparison between the observed and the fitted data is shown in figure 3.

From the estimated Weibull distribution we can obtain an estimation of the proportion of hidden consumers in interval k as:

$$F(\theta_k^*) - F(\theta_k), \quad k = 0, 1.$$

So, we compute a percentage of 24.26% hidden consumers in $q_0 = 30\,\mathrm{m}^3$ and a negligible percentage of hidden consumers in $q_1 = 70$.

Table 1. Estimated Parameters for the Weibull distribution

	θ_0^*	θ_1^*	ρ	μ
Estimated parameters (standard deviation)	0.9487×10^{-2} (1.21×10^{-5})	0.1840×10^{-1} (2.09×10^{-5})	0.9236×10^{-2} (1.04×10^{-5})	1.9012 (2.43×10^{-3})

Table 2. Correlation Coefficients Between Parameters Estimators

	θ_1^*	ρ	μ
θ_0^*	0.6801	0.6610	0.3015
θ_1^*		0.4834	0.3342
ρ			– 0.0211

5. Effects of Changing the Tariff

Once the true distribution of consumer types has been estimated, we now look at the equity and welfare effects of changing the tariff. In particular we introduce a two-part tariff which does not reduce current revenue levels and does not increase current consumption levels. Our interest in this case is that the possible welfare changes come from the reallocation of the current consumption and payments across consumers, which allows us to evaluate the ''equity'' of the increasing block rates across different types of consumers.[9]

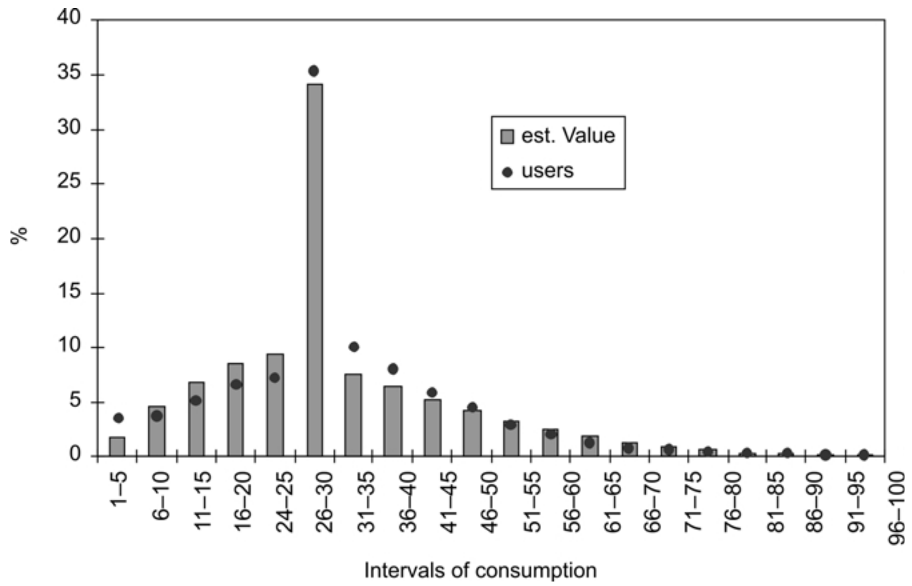

Figure 3. Observed and estimated frequencies.

If we suppose that the service is currently covering costs, the optimal two-part tariff will be one which maximizes the welfare of consumers, subject to the participation of all consumers, maintenance of the firm's current revenue levels and no increase in current levels of aggregate consumption.[10] This is equivalent to maximizing the expected surplus of the consumers subject to the firm's average revenue being greater than $\underline{\Pi}$, the firm's average profits, and consumption less than $\overline{Q}$, the average consumption, respectively.[11] Formally, the problem is expressed as

$$[P] \equiv \begin{cases} \max\limits_{\{A,p\}} & \displaystyle\int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{\theta}{2}(\alpha - p)^2 - A\right] f(\theta)\, d\theta \\ \text{s.t.} & \begin{cases} \dfrac{(\alpha - p)}{2} - A \geq 0 \\ A + (p - c)\ (\alpha - p) \displaystyle\int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)\, d\theta \geq \underline{\Pi} \\ (\alpha - p) \displaystyle\int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)\, d\theta \leq \overline{Q} \end{cases} \end{cases}$$

where $\int_p^\alpha \theta(\alpha - x)\, dx - A = \theta/2(\alpha - p)^2 - A$ is the net surplus of the consumer of type θ who is faced with the $T(q) = A + pq$ two-part tariff, $\alpha - p/2 - A$ is the surplus of the smallest consumer[12] and c is the marginal cost of the water.

Proposition 2: *The optimal two-part tariff is such that*

$$p^* = \begin{cases} \alpha - \frac{\overline{Q}}{E(\theta)} & \textit{if } c < c^* \\ \frac{\left[(\alpha + c)E(\theta) - \frac{1}{2}\right] - \sqrt{\left[(\alpha - c)E(\theta) + \frac{1}{2}\right]^2 2 - 4E(\theta)\underline{\Pi}}}{2E(\theta)} & \textit{if } c > c^*, \end{cases}$$

$$A^* = \begin{cases} \underline{\Pi} - \left(\alpha - \frac{\overline{Q}}{E(\theta)}\right)\overline{Q} + c\overline{Q} & \textit{if } c < c^* \\ \frac{\alpha - p}{2} & \textit{if } c > c^*, \end{cases}$$

9 It may also be of interest to look at an optimal two-part tariff that keeps revenue constant but allows more flexibility in consumption, and to introduce a capacity cost which takes into account the capacity constraint. However, we would need some type of time-of-use pricing to have a small probability that all people use water at the same time and that the system does not have the capacity to meet it. However, this is not possible with the available data.

10 The revenue constraint is equivalent to the constraint that the firm breaks even, under the assumption that the current tariff is doing this.

11 Actually, $\underline{\Pi}$ represents the firm's average profits net of average costs of suppling a level of consumption $\overline{Q}$

12 The smaller consumer is the one who buys one ''unit'' of water, defined as $\underline{\theta} = 1/(\alpha - p)$.

where c^* *is defined as the level of marginal cost* c *such that*

$$\alpha - \frac{\overline{Q}}{E(\theta)} = \frac{\left[(\alpha + c)E(\theta) - \frac{1}{2}\right] - \sqrt{2\left[(\alpha - c)E(\theta) + \frac{1}{2}\right]^2 - 4E(\theta)\underline{\Pi}}}{2E(\theta)},$$

and $E(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)$.

Proof: The maximization problem can be rewritten as

$$[P] \equiv \begin{cases} \max\limits_{\{A,p\}} & \dfrac{\theta}{2}(\alpha - p)^2 \; E(\theta) - A \\ \text{s.t.} & \begin{cases} \dfrac{(\alpha - p)}{2} - A \geq 0 \\ A + (p - c) \; (\alpha - p) \; E(\theta) \geq \underline{\Pi} \\ (\alpha - p) \; E(\theta) \leq \overline{Q} \end{cases} \end{cases}$$

where $E(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)$.

There exist two possible regions depending on whether the capacity constraint is binding or not:

1. For all the values of marginal cost c such that $p < \alpha - \overline{Q}/E\theta$, and then $Q(p) > \overline{Q}$, the optimal tariff is determined by the binding capacity constraint and by the binding revenue constraint, and so:

$$p^* = \alpha - \frac{\overline{Q}}{E(\theta)},$$
$$A^* = \underline{\Pi} - \left(\alpha - \frac{\overline{Q}}{E(\theta)}\right)\overline{Q} + c\overline{Q},$$

2. On the other hand, when the capacity constraint is not binding, that is to say, $Q(p) < \overline{Q}$, the optimal tariff is obtained by letting the participation constraint and the revenue constraint bind. From the first one,

$$A^* = \frac{\alpha - p}{2}, \tag{13}$$

and from the second one,

$$A^* + (p^* - c) \; (\alpha - p^*) \; E(\theta) = \underline{\Pi}. \tag{14}$$

Manipulating (13) and (14), we obtain:

$$E(\theta) \; p^2 - \left[(\alpha + c) \; E(\theta) - \frac{1}{2}\right] \; p - \left(\frac{\alpha}{2} - c \; \alpha \; E(\theta) + \underline{\Pi}\right) = 0,$$

Table 3. Values of the Parameters

	α	$\overline{Q}$	$\underline{\Pi}$	$\hat{E}(\theta)$	c^*
Value	4560	34.47	3083	0.0076485	29.16

so that:

$$p^* = \frac{\left[(\alpha + c)E(\theta) - \frac{1}{2}\right] - \sqrt{\left[(\alpha - c)E(\theta) + \frac{1}{2}\right]^2 2 - 4E(\theta)\underline{\Pi}}}{2E(\theta)}.$$

Then, we can define the critical value c^* as the value of the marginal cost c such that

$$\alpha - \frac{\overline{Q}}{E(\theta)} = \frac{\left[(\alpha + c)E(\theta) - \frac{1}{2}\right] - \sqrt{\left[(\alpha - c)E(\theta) + \frac{1}{2}\right]^2 2 - 4E(\theta)\underline{\Pi}}}{2E(\theta)}. \qquad \blacksquare$$

That is to say, c^* represents the value of marginal cost such that the optimal two-part tariff exactly induces an aggregate level of consumption equivalent to the current one. Table 3 presents the value of c^* given the values of the parameters α, $\overline{Q}$, $\underline{\Pi}$ and $\hat{E}(\theta)$.[13] Figure 4 shows the optimal tariff for each value of c given those values of the parameters.

In the literature, the value of the water marginal cost is estimated to be smaller than c^* (see, for example, Mercer and Morgan 1985 and Renzetti 1992).[14] Therefore, the consumption constraint is binding and the solution for p and A with $c < c^*$ applies.

To evaluate the welfare changes we impose the constraint that the firm does not modify the aggregate revenue level, and so we can write

$$\overline{T} = A + p\overline{Q},$$

13 To evaluate the optimal tariff it is necessary to previously estimate $E(\theta)$. This can be done by two statistical methods essentially independent. The first one only uses the values θ_0^* and θ_1^* obtained from the estimation of the system of equations. To take the expected value of the discrete version of the random variable θ is enough. With this method, therefore, we incorporate the information provided by the system's theoretical constraints. The second estimation, however, uses the adjusted Weibull distribution and, therefore, it is based on disaggregated data in various subintervals within each interval of consumption, including no information about the area of the city. With the aim of combining all the available information given by the two different sources, we take the average of the two estimations as the final estimator of $E(\theta)$, $\hat{E}(\theta)$. If the estimation processes and the original data were really independent, the variance of the estimator created by averaging the previous two would be a quarter of the sum of the variances of them.

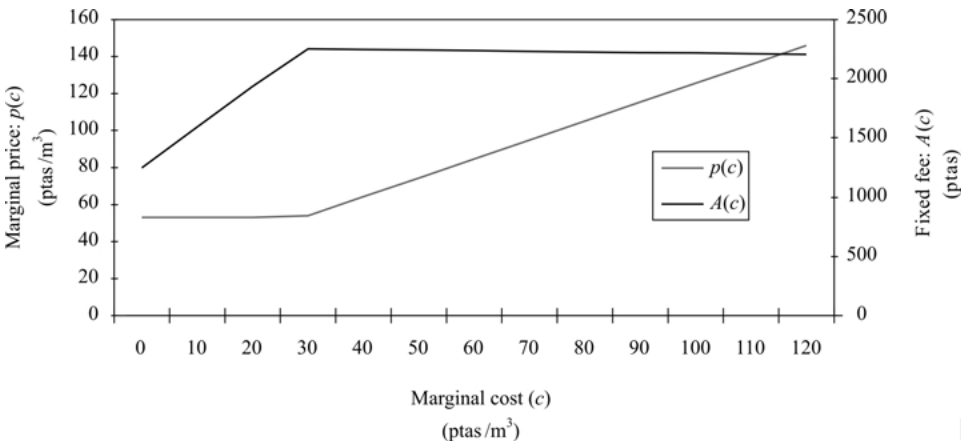

Figure 4. The optimal tariff for each value of marginal cost.

where $\overline{T}$ are the average revenue per consumer obtained by the firm with the current tariffs (equal to 3083 pesetas).

This condition allows us to obtain the optimal two-part tariff (when $c < c^*$) as

$$p^* = \alpha - \frac{\overline{Q}}{E(\theta)}, \qquad (15)$$
$$A^* = \overline{T} - \left(\alpha - \frac{\overline{Q}}{E(\theta)}\right)\overline{Q},$$

where the fixed fee does not depend on the value of c, because at the optimum the following must be satisfied:[15]

$$A^* + p\overline{Q} = \underline{\Pi} + c\overline{Q}.$$

Table 4 puts forward the average bills, the average consumption levels and the percentage of users who are currently in each interval. The last row sketches the average consumption $\overline{Q}$ and the average bill per consumer $\underline{\Pi}$. Given $\hat{E}(\theta)$ and using the data for $\overline{Q}$ and $\overline{T}$, we evaluate (15) and obtain $p^* = 53.2$ and $A^* = 1248$.

The new tariff changes the distribution of consumption. As can be seen in figure 5 a

14 In this analysis we use values of water marginal cost which have been estimated for the U.S. and Canada since no such estimates are available for Spain.

15 We are implicitly defining the firm's profit ranges (per consumer) because $\underline{\Pi} + c\overline{Q} = 3083$, and if $c = 0$, then $\underline{\Pi} = 3083$, and if $c = c^* = 29, 16$, then $\underline{\Pi} = 2078$.

Table 4. Users, Average Revenues and Average Consumption Levels in each Interval of Consumption of the Current Tariff

Section	% Users	$\overline{x}$	$\overline{T}$
0	61.61	23.67	2280
1	35.33	43.34	3475
2	2.66	97.02	8521
3	0.40	500.30	55953
Total	100.00	$\overline{Q} = 34.47$	$\underline{\Pi} = 3083$

two-part tariff generates a smooth distribution with less consumers consuming 30 m^3 and more in the other intervals.[16]

5.1. Welfare analysis

In order to evaluate the welfare changes we have to calculate the individual net surplus for each tariff, the current one (C) and the new one (N), and then compare them. In the case of the current tariff it is necessary to distinguish between the hidden individuals and the

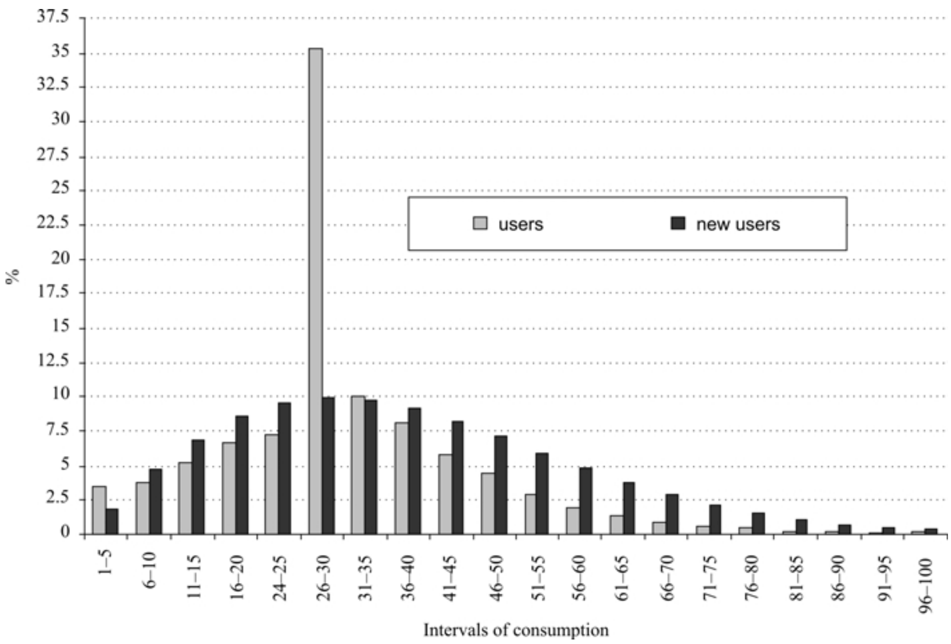

Figure 5. Changes in the distribution of consumption.

Table 5. Values to Calculate the Consumer Surplus

Non Hidden	p_C^k	A_C^k
0	0.0	2280
1	70.5	420
2	83.5	420
3	111.0	420
Hidden	q_C^k	T_C^k
0	30	2280
2	70	5355
3	200	17120

non-hidden ones. For the non-hidden individuals, the consumer's surplus of type θ is defined as

$$CS_C^{NH}(\theta) = \int_{p_C^k}^{\alpha} \theta(\alpha - p)dp - A_C^k = \frac{\theta}{2}\left(\alpha - p_C^k\right)^2 - A_C^k,$$

where p_C^k and A_C^k are respectively the price and the fixed fee for each of the intervals of consumption $k = 0, 1, 2, 3$. On the other hand, the surplus for hidden individuals must be defined depending on the quantity:

$$CS_C^H(\theta) = q_k\left(\alpha - \frac{1}{2\theta} q_k\right) - T_C^k,$$

where q_C^k and T_C^k are the associated quantities and the payments for the intervals with ''hidden'' individuals ($k = 0, 1, 2$). Table 5 presents the values of the parameters that we use to evaluate the consumer surplus. The surplus associated to the new tariff is written for all consumers as

$$CS_N(\theta) = \frac{\theta}{2}(\alpha - p_N)^2 - A_N(c),$$

where $p_N = 53.2$ and $A_N(c) = 1248$ are the prices for the new tariff.

In this way, the increase in individual welfare derived from the change in tariff is

16 The consumption distribution with the new tariff is defined as:

$$F_q(p) = F_\theta\left(\frac{q}{\alpha - p}\right),$$

with $\alpha = 4560$ and $p = 53.2$.

Table 6. Welfare Changes Implied by the New Tariff

Type Interval $[\theta_i, \theta_{i+1}] \times 10^{-4}$	Frequency	Current Tariff Consumption $[q_i, q_{i+1}]$	New Tariff Consumption $[q_i, q_{i+1}]$	ΔW
$[0, 42.79]$	19.76	$[0, 19]$	$[0, 18.8]$	$(+76)$
$[42.79, 65.79]$	21.06	$[19, 30]$	$[18.8, 29]$	(-57)
$[65.79, 72.93]$	6.35	$[30, 30]$	$[29, 33]$	(-25)
$[72.93, 94.87]$	17.91	$[30, 30]$	$[33, 42]$	$(+584)$
$[94.87, 106.40]$	7.89	$[30, 35]$	$[43, 48]$	$(+4)$
$[106.40, \overline{\theta}]$	27.01	> 35	> 48	$(+152)$

defined as the difference in surpluses $\Delta W(\theta) = CS_N(\theta) - CS_O(\theta) - c\overline{Q}$, which is written as follows for the different intervals of consumers:

$$[\Delta W] \equiv \begin{cases} 1032 - 241176.88\theta & \theta \in [0, 0.006579] \\ 135768 - 10155623\theta - 450/\theta & \theta \in [0.006579, 0.009487] \\ 77817.99\theta - 828 & \theta \in [0.009487, 0.015590] \\ 10155623\theta + 2450/\theta - 315093 & \theta \in [0.015590, 0.018390] \\ 136096.98\theta - 828 & \theta \in [0.018390, 0.044670] \\ 258822.6\theta - 828 & \theta \in [0.044670, \overline{\theta}]. \end{cases}$$

In table 6 we present the welfare changes implied by the new tariff. We classify the consumers in different intervals according to whether they have improved themselves or worsened with the new tariff.[17] Each interval of consumption is defined by the upper and lower parameters and by their associated consumption with the current tariff.[18] Likewise, we calculate the proportion of individuals for each interval, which allows us to conclude that 65.6% of individuals increase their welfare while the remaining 34.4% decrease it. The consumers who were previously consuming a quantity less than 20 cubic meters are better off with the new tariff (they consume less but also pay less than before). Concerning the consumers who expand their consumption, only those who were located between 30 and 35 m^3 worsen (7.9%), as the improvements derived from a reduction in the marginal price do not cover the increase experimented by the fixed fee.

Finally, we calculate the aggregate surplus by using the equation $\sum_{i=1}^{6} f_i \, \Delta W_i(\theta)$, where $\Delta W_i(\cdot)$ is the increase in the welfare obtained by the individuals belonging to the ith interval, limited by θ_i and θ_{i+1}, and f_i is the frequency of that interval. The total welfare is increased meaning that the introduction of the new tariff implies an efficiency improvement. Since the new tariff does not imply an improvement for all the individuals, this result shows that the subsidies conceded to some consumers under the current tariff are eliminated with the new one.

17 The values of the surplus for the different intervals have been evaluated by numerical integration methods using the Weibull density function estimated in section 5.

18 To obtain the intervals we find the roots of ΔW.

Table 7. Frequency, Payment and Average Price

	Frequency			Payment		Average Price	
Interval	Current	New	q	Current	New	Current	New
1–5	3.5	1.76	3	2280	1408	760	469
6–10	3.7	4.67	8	2280	1674	285	209
11–15	5.15	6.95	13	2280	1940	175	149
16–20	6.63	8.6	18	2280	2206	127	123
21–25	7.27	9.59	23	2280	2472	99	107
26–30	35.36	9.95	28	2280	2738	81	98
31–35	10.03	9.77	33	2747	3004	83	91
36–40	8.09	9.14	38	3099	3270	82	86
41–45	5.85	8.21	43	3452	3536	80	82
46–50	4.44	7.11	48	3804	3802	79	79
51–55	2.86	5.95	53	4157	4068	78	77
56–60	1.95	4.82	58	4509	4334	78	75
61–65	1.29	3.78	63	4862	4600	77	73
66–70	0.81	2.88	68	5214	4866	77	72
71–75	0.64	2.14	73	6516	5132	89	70
76–80	0.47	1.54	78	6933	5398	89	69
81–85	0.27	1.08	83	7351	5664	89	68
86–90	0.24	0.74	88	7768	5930	88	67
91–95	0.17	0.49	93	8186	6196	88	67

5.2. Equity

If income and water use are positively correlated, an increasing block rate pricing can improve interpersonal equity. However, when this pricing schedule includes a fixed quota with a minimum free allowance, the low income-low volume users could be worse off than with a two-part tariff, because the introduction of the two-part tariff implies a reduction in the fixed fee for all consumers in the interval of consumption $[0, 30]$.

In order to compare the equity between the current tariff and the new one, we analyze the payments and average prices associated with both tariffs. As can be observed in table 7, the introduction of a two-part tariff reduces the payments and average prices of the consumers with levels of consumption between 1 and 20 and of more than 48 cubic meters.

6. Conclusions

In this paper we show that when an increasing block rates pricing including a minimum charge is used, the estimation of the demand is biased if we use the distribution of the observed consumption as a proxy for the distribution of consumer preferences because there are different types of customers with the same consumption level. We present a two-step method to estimate the demand in this situation. First, we estimate the demand relation for each block as a function where the price variables are the marginal price of that block and the marginal prices of the adjacent blocks. This allows us to write the observed distribution of consumption as a function of the theoretical distribution of consumers induced by the current tariff. Second, we use the observed frequencies of consumption to estimate the distribution parameters. This allows us to obtain an estimation of the number of consumers induced to demand the same amount of water, the ''hidden'' consumers.

Once the distribution of consumer types has been estimated, we look at the equity and welfare effects of changing the tariff. In particular, we introduce a two-part tariff which does not reduce current revenue levels and does not increase current consumption levels. Thus, we show that the introduction of this two-part tariff improves the welfare of consumers who were consuming a quantity lower than 20 m^3 and higher than 40 m^3. The consumers who are located between 30 and 45 m^3 worsen since the improvements derived from a reduction in the marginal price do not cover the increase experimented by the fixed fee. We also contend that in aggregate terms total welfare is higher with the new tariff.

We are aware that a two-part tariff does not achieve all of the potential gains which could be achieved with more sophisticated tariffs. For example, a pricing schedule including a two-part tariff for consumption lower than 200 m^3, and the current tariff for consumption higher than 200 m^3, could obtain higher equity and efficiency levels. In any case, we would need a more intensive data set to compute an optimal multi-part tariffs.

Our results show that increasing block pricing is an inferior pricing tool and should be abandoned. As a referee pointed, this raises the question of why regulators recommended this practice for so long. One possible story (as table 7 suggests) is that the mid-levels consumers—which would be worse off under the two-part tariff have very strong political influence (maybe because they are the median voters). To prove this intuition we need to build a model where the consumers choose the tariff, but this is another story.

Appendix

A.1. System's Constraints

Differentiating each demand function with respect to prices, we have

$$\begin{aligned}
\frac{\partial D_0(p_0,p_1)}{\partial p_0} &= q_0 f(\theta_0^*)\frac{d\theta_0^*}{dp_0} < 0,\\
\frac{\partial D_k(p_{k-1},p_k,p_{k+1})}{\partial p_k} &= \int_{\theta_{k-1}^*(p_k)}^{\theta_k} -\theta f(\theta)\,d\theta - \theta_{k-1}^*(\alpha-p_k)f(\theta_{k-1}^*)\frac{d\theta_{k-1}^*}{dp_k} + q_k f(\theta_k^*)\frac{d\theta_k^*}{dp_k} < 0,\\
\frac{\partial D_3(p_2,p_3)}{\partial p_3} &= \int_{\theta_1^*(p_1,p_2)}^{\bar{\theta}} -\theta f(\theta)\,d\theta - \theta_2^*(\alpha-p_3)f(\theta_2^*)\frac{d\theta_2^*}{dp_3} < 0,
\end{aligned}$$

$k = 1, 2$. To obtain the sign of the partial derivatives we have differentiated the marginal consumers with respect to the prices. So, the indifferent consumers can be implicitly defined as

$$U(q_k,\theta_k^*) - p_k q_k = U(\theta_k^*(\alpha-p_{k+1}),\theta_k^*) - p_{k+1}\theta_k^*(\alpha-p_{k+1}).$$

Differentiating this expression with respect to the prices we obtain

$$\frac{d\theta_k^*(p_k,p_{k+1})}{dp_k} = \frac{q_k}{A} < 0, \quad \frac{d\theta_k^*(p_k,p_{k+1})}{dp_{k+1}} = -\frac{\theta_k^*(\alpha p_{k+1})}{A} > 0,$$

where $A = U'_\theta(q_k, \theta^*_k) - U'_\theta(\theta^*_k(\alpha - p_{k+1}), \theta^*_k) < 0$. Furthermore, we can write that

$$f(\theta^*_k)\frac{d\theta^*_k}{dp_k} = -\frac{\partial D_{k+1}/\partial p_k}{\theta^*_k(\alpha - p_{k+1})}, \quad k = 0, 1, 2,$$
$$f(\theta^*_k)\frac{d\theta^*_k}{dp_{k+1}} = \frac{\partial D_k/\partial p_{k+1}}{q_k}, \quad k = 0, 1, 2,$$

and, therefore, derive that for $k = 0, 1, 2, \partial D_{k+1}/\partial p_k > 0$ and $\partial D_k/\partial p_{k+1} > 0$. To prove the symmetry of the estimated parameters, we must note that $\partial D_k/\partial p_{k+1} = q_k f(\theta^*_k) d\theta^*_k/dp_{k+1}$. Differentiating the indifference condition of the marginal consumer θ^*_k with respect to prices p_k and p_{k+1} we can write that $d\theta^*_k/dp_{k+1} = -\theta^*_k(\alpha - p_{k+1})/A$ where $A = U'_\theta(q_k, \theta^*_k) - U'_\theta(\theta^*_k(\alpha - p_{k+1}), \theta^*_k)$. As $d\theta^*_k/p_k = q_k/A$, we conclude that

$$\frac{\partial D_k}{\partial p_{k+1}} = -\theta^*_k(\alpha - p_{k+1})f(\theta^*_k)\frac{d\theta^*_k}{dp_k} = \frac{\partial D_{k+1}}{\partial p_k}.$$

If $\beta ij = \partial Di/\partial p_j$, thus $\beta_{01} = \beta_{10}$ $\beta_{12} = \beta_{21}$ and $\beta_{23} = \beta_{32}$. On the other hand, by substituting the cross derivatives in direct derivatives, we can write

$$\frac{\partial D_1}{\partial p_1} + \delta_0\frac{\partial D_0}{\partial p_1} + \frac{1}{\delta_1}\frac{\partial D_2}{\partial p_1} = \int_{\theta^*_0(p_0,p_1)}^{\theta_1} -\theta f(\theta)\,d\theta = K_1, \tag{16}$$
$$\frac{\partial D_2}{\partial p_2} + \delta_1\frac{\partial D_1}{\partial p_2} + \frac{1}{\delta_2}\frac{\partial D_3}{\partial p_2} = \int_{\theta^*_1(p_1,p_2)}^{\theta_2} -\theta f(\theta)\,d\theta = K_2, \tag{17}$$
$$\frac{\partial D_3}{\partial p_3} + \delta_2\frac{\partial D_2}{\partial p_3} = \int_{\theta^*_2(p_2,p_3)}^{\bar{\theta}} -\theta f(\theta)\,d\theta = K_3, \tag{18}$$

with $\delta_k = \theta^*_k(\alpha - p_{k+1})/q_k$, $k = 0, 1, 2$, where δ_0, δ_1 and δ_2 are the changes in the consumption of the marginal individuals when they move from interval zero to one, from one to two and from two to three. Then, we can write K_j, $j = 1, 2$, using the estimated parameters, as $K_j = \beta_{j,j} + \delta_{j-1}\beta_{j-1,j} + 1/\delta_j\beta_{j+1,j}$.

A.2. Data

A.2.1. Average Consumption Levels and Effective Prices in each Interval and Zone ($\bar{x}^t_k$ and p^t_k)

Zone	$\bar{x}_0$	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	p_0	p_1	p_2	p_3
I	22.2368	44.0353	98.5500	403.4333	55.57	63.42	67.63	91.29
II	24.3208	42.8175	96.6667	1019.2468	61.23	69.86	79.06	114.54
III	23.4708	41.7155	99.2732	366.3312	61.77	70.35	82.87	115.87
IV	22.6810	46.2906	108.1861	372.3163	61.42	69.63	80.17	106.99
V	22.8508	45.2314	101.4405	1456.2324	60.15	67.65	78.19	111.14
VI	22.2332	46.2563	94.3733	514.2123	44.22	49.01	58.06	97.21
VII	22.5133	45.6658	95.3062	305.1098	59.65	67.75	80.07	105.38

A.2.2. Proportion of Collected Sewage in each Interval and Zone (γ_k^t)

	Zones						
Interval	I	II	III	IV	V	VI	VII
0	73.21	96.79	99.06	97.59	92.28	25.91	90.19
31	73.79	97.63	99.44	96.76	89.44	20.42	89.81
71	48.82	85.69	97.96	89.27	82.86	17.93	88.94
201	34.97	96.15	99.65	76.28	87.20	50.55	72.06

A.2.3. Average Prices and Consumptions Levels

Interval (m^3)	Supply	Sewage	p_k = Efective Price	$\bar{x}$ = Average Consumption
0–30	38 pts	24 pts	62.0 pts	23.67 m^3
31–70	43.5 pts	27 pts	70.5 pts	43.34 m^3
71–200	52.5 pts	31 pts	83.5 pts	97.02 m^3
+200	78 pts	38 pts	111.0 pts	500.03 m^3

A.2.4. Frequencies

Interval	Percentage Subscribers	$\sum$ Percentage Subscribers	Percentage Users	$\sum$ Percentage Users
1–5	4.59	4.59	3.50	3.50
6–10	4.67	9.26	3.70	7.19
11–15	6.09	15.34	5.15	12.34
16–20	7.29	22.63	6.63	18.97
21–25	7.38	30.00	7.27	26.25
26–30	38.02	68.02	35.36	61.61
31–35	8.40	76.42	10.03	71.64
36–40	6.39	82.81	8.09	79.73
41–45	4.56	87.37	5.85	85.58
46–50	3.29	90.67	4.44	90.02
51–55	2.27	92.94	2.86	92.88
56–60	1.68	94.62	1.95	94.83
61–65	1.13	95.75	1.29	96.12
66–70	0.81	96.56	0.81	96.94
71–75	0.60	97.16	0.64	97.58
76–80	0.47	97.64	0.47	98.05
81–85	0.34	97.97	0.27	98.32
86–90	0.29	98.26	0.24	98.56
91–95	0.21	98.47	0.17	98.73
96–100	0.18	98.65	0.19	98.92
101–200	0.16	98.81	0.13	99.06

Subscribers represent the number of meters and users represent the number of consumers associated to each meter.

A.3. Estimation of $F(\theta)$

	Sample Value	Estimated Value		Sample Value	Estimated Value
1	3.50	1.714	12	1.95	2.45
2	3.70	4.562	13	1.29	1.78
3	5.11	6.825	14	0.81	1.26
4	6.63	8.472	15	0.64	0.874
5	7.27	9.484	16	0.47	0.587
6	35.36	34.069	17	0.27	0.385
7	10.03	7.624	18	0.24	0.245
8	8.09	6.473	19	0.17	0.152
9	5.85	5.316	20	0.19	0.0927
10	4.44	4.230	21	0.13	0.1255
11	2.86	3.26			

A.4. Map of the City

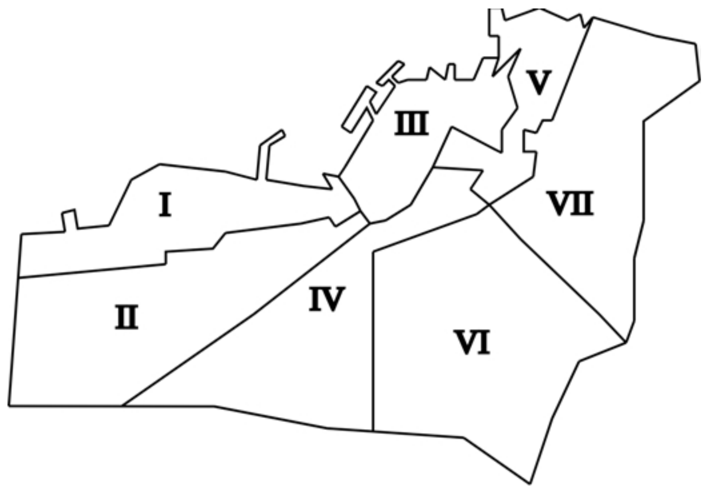

Figure 6. Map of the city.

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