# The Veto Mechanism Revisited[*]

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**Abstract.** It is difficult to argue that coalition formation is costless and free. In this paper, only a subset of the set of all possible coalitions in an economy or a game, is considered to be really formed. The consequences that such restriction has on the veto mechanism are analyzed. The restricted veto mechanism is extended to the pondered veto mechanism with rates of participation of the the players. It is shown that it is enough to consider the veto power of a subset $\mathcal{S}$ of coalitions, which differs from the set of all coalitions, in order to obtain the Walrasian allocations or, alternatively, the Edgeworth equilibria. In particular, it is shown that the pondered veto power, with rates of participation arbitrarily close to one, of only one coalition, namely, the coalition of all agents, blocks any non Walrasian allocation.

**Keywords:** Coalitions, core, fuzzy core, Edgeworth equilibrium, exchange economy, continuum economy, Walrasian equilibrium.

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# 1 Introduction

The core of an economy consists of those feasible allocations which no coalition of agents can block. Checking whether a given allocation belongs to the core seems to require to look upon the whole set of possible coalitions in order to test whether any group of agents, by using their own resources, can improve upon such allocation. Unless the economy is extremely small, this may be a great task. We also remark that the formation of coalitions may imply some theoretical difficulties. In fact, it is usually argued that the costs, which arise from forming a coalition, are not at all negligible. Moreover, players will form a coalition only if they know each other. Incompatibilities among different agents may arise and a big amount of information and communication might be needed to really form a coalition. Thus, sometimes, it will not suffice to say merely that several agents or players constitute a coalition. The fact that they are organized in some way, and perhaps they are not entirely free agents, may result in high formation costs, commitments and constraints, which make difficult to assume that the veto mechanism, leading to the core, works freely and spontaneously.

In the case of continuum, or atomless, economies the size of a coalition $S$ is given by the measure considered in the agent space. The size or the measure of a coalition may have different meanings. For example, following Schmeidler (1972), we can interpret the measure of a coalition $S$ as the amount of (or cost of) information and communication needed in order to form the coalition $S$. Then, may be meaningful to consider those coalitions whose size converges to zero or, symetrically, whose size converges to one; that is, the coalitions that do not involve high costs to be formed. Given the relative value that a measure may have, it may also be interesting to consider only coalitions with identical size.

In this article, the difficulty to argue that coalition formation is costless leads us to consider a restricted veto mechanism. We assume that only a subset $\mathcal{S}$ of the set of all possible coalitions in an economy is the set of admissible coalitions and we study the consequences that this assumption has with regard to the veto mechanism.

The coalitional veto mechanism leads to the cooperative solution of the core and, in economies with $n$-agents, to the concept of Edgeworth equilibrium (feasible allocations which are not blocked in any replicated economy). If only some coalitions, those belonging to a subset $\mathcal{S}$ of the whole set of possible coalitions, are allowed to be formed, then we can define the $\mathcal{S}$-core concept. Similarly, if the set of coalitions is restricted in each $r$-replica economy to a subset $\mathcal{S}(r)$ of admissible coalitions, then we can define the concept of $\mathcal{S}(r)$-Edgeworth equilibrium of an economy $\mathcal{E}$.

If the definition of coalitions is enlarged in order to allow a participation of the agents with a rate belonging to the rational interval $[0, 1]$ and if the preferences are convex, an Edgeworth equilibrium can also be defined as an attainable which can not be blocked by a coalition with rational rates of participation. A fuzzy coalition is a coalition whose rates of participation can take any value in the real interval $[0, 1]$. The fuzzy core is the set of all attainable allocations which cannot be blocked by a fuzzy coalition (see Aubin (1979)). Restricting attention to a subset of admissible fuzzy coalitions makes sense only if the participation rate of each agent is required to be strictly positive. Note that, if it is not so, then any subcoalition of an admissible fuzzy coalition would also be admissible. Even more, if null participation rates are allowed and a fuzzy coalition $S$ blocks an allocation, then this allocation is also blocked by any fuzzy coalition containing $S$. Thus, in order to state a meaningful extension of the restricted veto mechanism to the veto system proposed by Aubin, all participation rates are required to take any value in the real interval $(0, 1]$. Precisely, we call pondered veto to the veto with strictly positive participation rates. Then, we define and analyze the notion of $\mathcal{S}$-pondered core, by restricting the set of blocking coalitions to the set of admissible coalitions $\mathcal{S}$.

In section 2,we analyze the restricted veto mechanism and we define the concept of $\mathcal{S}$-core. Different specifications of the set $\mathcal{S}$ of admissible coalitions, allow us to interpret the results in Schmeidler (1972), Grodal (1972) and Vind (1972) in terms of $\mathcal{S}$-cores. The $\mathcal{S}(r)$-Edgeworth equilibrium and $\mathcal{S}$-Pondered core are defined in section 3. Following García and Hervés (1993), we interpret a continuum economy with $n$ types of agents as an economy with $n$ agents. In this way, we extend and generalize some results in Florenzano (1990) and Hüsseinov (1994), while obtaining other new characterizations of the core and Edgeworth equilibrium. We make clear the power of the pondered veto mechanism showing that it is enough to consider the coalition of all agents as an admissible coalition, in order to guarantee that the only allocations which are not blocked by the pondered veto system are just the Walrasian allocations. Therefore, if the coalition of all agents is an admissible coalition, the restricted pondered veto mechanism blocks those allocations which are not decentralizable by a price system. We conclude, in section 4, with some comments and final remarks.

# 2 The restricted veto mechanism

Let the exchange economy $\mathcal{E} = (X, (I, \mathcal{A}, \mu), \omega(t), \preceq_t, t \in I)$ where an ordered Banach space $X$ is the commodity space, $(I, \mathcal{A}, \mu)$ is a finite measure space, being $I$ the set of agents and $\mu$ a measure on the $\sigma$-algebra $\mathcal{A}$. Each agent $t \in I$ is characterized by her preference relation $\preceq_t$ on her consumption set $X_+$ and her initial endowment $\omega(t) \in X_+$. The function $\omega : I \to X_+$, which associates to each agent her initial endowments, is Bochner-integrable. The function $\preceq$, which associates to each agent her preferences is measurable. An allocation is a Bochner-integrable function $f : I \to X_+$ . An allocation $f$ is feasible if $\int_I f(t) d\mu(t) \leq \int_I \omega(t) d\mu(t)$.

A coalition $S$ is a measurable subset of $I$, such that $\mu(S) > 0$. Given $S \in \mathcal{A}$, $\mu(S)$ represents the size of the coalition $S$. The coalition $S$ improves upon or blocks an allocation $f$ via $g$ if $\int_S g(t)d\mu(t) \leq \int_S \omega(t)d\mu(t)$ and $g(t) \succ_t f(t)$ for almost all $t \in S$. A feasible allocation $f$ belongs to the core of the economy $\mathcal{E}$ if it cannot be blocked by any coalition $S \in \mathcal{A}$. Let $C(\mathcal{E})$ denote the core of the economy $\mathcal{E}$.

Note that if $I = \{1, \ldots, n\}$, then $\mathcal{A} = \mathcal{P}(I)$ is the set of all subsets of $I$, and $\mu(S)$ denotes the cardinal of the coalition $S \in \mathcal{A}$. If we consider an atomless economy, then $I = [0, 1]$, $\mathcal{A}$ is the $\sigma$-algebra of measurable subsets of $I$ and $\mu$ is the Lebesgue measure. We can also consider mixed economies; in this case $I = [0, 1] \bigcup \{1, \ldots, n\}$, $\mathcal{A}$ is the product $\sigma$-algebra of measurable subsets $[0, 1]$ and subsets of $\{1, \ldots, n\}$, and $\mu$ is the product measure.

Restricting the set of coalitions to be considered to a subset $\mathcal{S}$ of admissible coalitions, leads us to introduce the cooperative solution that we call $\mathcal{S}$-core. A feasible allocation belongs to the $\mathcal{S}$-core of an economy $\mathcal{E}$ if it is not blocked by any admissible coalitions. Next we formalize the $\mathcal{S}$-core concept.

**Definition** 2.1 *Let $\mathcal{S} \subseteq \mathcal{A}$, with $\mu(S) > 0$ for every $S \in \mathcal{S}$. A feasible allocation $f$ of the economy $\mathcal{E}$ belongs to the $\mathcal{S}$-core of $\mathcal{E}$, and we denote $f \in \mathcal{S}$-$C(\mathcal{E})$, if it is not blocked by any coalition $S \in \mathcal{S}$.*

Note that the concept of $\mathcal{S}$-core generalizes the notion of core of an economy. If $\mathcal{S} = \mathcal{A}$, then the $\mathcal{S}$-core becomes the core.

From the definition of $\mathcal{S}$-core one can easily infer the following properties:

(P.1) Let $\mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{A}$, with $\mathcal{S}_1 \subset \mathcal{S}_2$. Then $\mathcal{S}_2$-$C(\mathcal{E}) \subset \mathcal{S}_1$-$C(\mathcal{E})$ . In particular, $C(\mathcal{E}) \subseteq \mathcal{S}$-$C(\mathcal{E})$ whatever $\mathcal{S} \subseteq \mathcal{A}$ may be.

(P.2) $\mathcal{S}_1$-$C(\mathcal{E}) \bigcap \mathcal{S}_2$-$C(\mathcal{E}) = (\mathcal{S}_1 \bigcup \mathcal{S}_2)$-$C(\mathcal{E})$ , for every $\mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{A}$.

From (P.1) it is deduced that if the core is non empty, then so is the $\mathcal{S}$-core, for every set $\mathcal{S}$ of admissible coalitions. (P.2) implies that if $\mathcal{A} = \bigcup_i \mathcal{S}_i$, then $\bigcap_i (\mathcal{S}_i$-$C(\mathcal{E})\ ) = C(\mathcal{E})$ . That is, if $\mathcal{P}$ is any partition of the whole coalition set $\mathcal{A}$, then the allocations belonging to the core are those allocations that belong to every $\mathcal{S}$-core, with $\mathcal{S} \in \mathcal{P}$. In particular, the intersection of the $\mathcal{S}$-cores of a partition $\mathcal{P}$ does not depend on $\mathcal{P}$.

In the remainder of this section we consider an atomless economy

$$\mathcal{E} = ((I, \mathcal{A}, \mu), \omega(t), \preceq_t, t \in I)$$

with $\mathbb{R}^\ell$ as commodity space. The space $(I, \mathcal{A}, \mu)$, which represents the set of agents, is an atomless measure space. For example, $I$ denotes the real interval $[0, 1]$, $\mathcal{A}$ is the $\sigma$-algebra of measurable subsets of $I$ and $\mu$ is the Lebesgue measure.

Schmeidler (1972) shows that blocking only with small coalitions is enough to get the Pareto optimality and the core-Walras equivalence, in atomless economies with a finite number of commodities. The restricted veto mechanism setting allows to formalize this result in terms of $\mathcal{S}$-cores. For this, given a measure $\nu$, absolutely continuous with respect to $\mu$., and $\varepsilon > 0$, let $\mathcal{S}_\varepsilon^\nu = \{S \in \mathcal{A} | \nu(S) \leq \varepsilon\}$, and $\mathcal{S}_\varepsilon = \{S \in \mathcal{A} | \mu(S) = \varepsilon\}$. Then, we have: $\mathcal{S}_\varepsilon^\nu$-$C(\mathcal{E}) = C(\mathcal{E})$, for every $\varepsilon > 0$. and there exists $\varepsilon_0 > 0$, such that $\mathcal{S}_\varepsilon$-$C(\mathcal{E})$ =$C(\mathcal{E})$, for every $\varepsilon \leq \varepsilon_0$.

Grodal (1972) obtains a further substantial restriction in the coalitions that are allowed to form and still gets the core-Walras equivalence. She shows that the small coalitions can be chosen in a neighborhood of $\ell$ points in the space of agents, being $\ell$ the number of commodities to be exchange in the market. Next we include this result in our setting. For this, given $\varepsilon > 0$, and a positive integer $k$, let $\mathcal{S}_\varepsilon^k = \{S \in \mathcal{A} | S = \bigcup_{i=1}^k S_i$, with $diam(S_i) \leq \varepsilon$ and $\mu(S_i) \leq \varepsilon\}$. The result in Grodal (1972) can be written as follows: There exists $\varepsilon_0 > 0$, such that $\mathcal{S}_\varepsilon^{\ell+1}$-$C(\mathcal{E}) = C(\mathcal{E})$, for every $\varepsilon \leq \varepsilon_0$. Moreover, if preferences are convex and weakly monotone, then $\ell + 1$ can be replaced by $\ell$.

Vind (1972) shows that if an allocation does not belong to the core of a continuum economy, then for any $\varepsilon$, with $0 < \varepsilon < 1$, there exists a coalition $S$, with $\mu(S) = \varepsilon$, blocking such allocation. In particular, the veto power of arbitrarily big coalitions is enough to get the core. So, if the allocation is determined by a vote system among the traders, the only allocations for which there is no coalition voting for some other allocation are the Walrasian allocations. Vind's (1972) results can also be formulated, within the restricted veto mechanism framework, in terms of $\mathcal{S}$-cores. Precisely, if for almost all $t \in I$ the preference relation $\succ_t$ is continuous, monotone and measurable, then $\mathcal{S}_\varepsilon$-$C(\mathcal{E}) = C(\mathcal{E})$, for every $\varepsilon$, such that $0 < \varepsilon < 1$.

We observe that the restricted veto mechanism allows to include Schmeidler, Grodal and Vind's result within a same context in terms of $\mathcal{S}$-cores. The coincidence results between the core and $\mathcal{S}$-core of atomless economies show that the equivalence core-Walras does still hold by considering the restricted veto mechanism.

# 3 The pondered veto mechanism

The restricted veto mechanism leads also to introduce the concept of $\mathcal{S}(r)$-Edgeworth equilibrium. For this, consider an exchange economy $\mathcal{E}_n$ with $n$ agents. For each positive integer $r$, the $r$-fold replica economy of $\mathcal{E}_n$ is a new exchange economy with $rn$ agents, indexed by $ij$, $i = 1, \ldots, n$; $j = 1, \ldots, r$ such that the consumer $ij$ is characterized by the initial endowment $\omega_{ij} = \omega_i$ and the preference relation $\succeq_{ij} = \succeq_i$ . Every allocation $x = (x_1, \ldots, x_n)$ of $\mathcal{E}_n$ can be considered as an allocation $rx = (x_{11}, \ldots, x_{1r}, \ldots, x_{n1}, \ldots, x_{nr})$ in the $r$-fold replica economy $r\mathcal{E}_n$ , where $x_{ij} = x_i$, for $j = 1, \ldots, r$ and $i = 1, \ldots, n$. The allocation $rx$ is called equal treatment allocation. Let $C'(r\mathcal{E}_n)$ denote the set of allocations $x$ for $\mathcal{E}_n$, such that $rx \in C(r\mathcal{E}_n)$. An Edgeworth equilibrium for the economy $\mathcal{E}_n$ is a feasible allocation $x = (x_1, \ldots, x_n)$ such that $rx$ belongs to the core of $r\mathcal{E}_n$ for every $r$. Let $E(\mathcal{E}_n)$ denote the set of Edgeworth equilibria. That is, $E(\mathcal{E}_n) = \bigcap_{r \geq 1} C'(r\mathcal{E}_n)$.

Now, for each positive integer $r$, let us consider a subset $\mathcal{S}(r)$ of the set of all coalitions in the replica economy $r\mathcal{E}_n$ . We will refer to $\mathcal{S}(r)$ as the set of admissible coalitions in the economy $r\mathcal{E}_n$ .

**Definition** 3.1 *A $\mathcal{S}(r)$-Edgeworth equilibrium for the economy $\mathcal{E}_n$ is a feasible allocation $x = (x_1, \ldots, x_n)$, such that $rx \in \mathcal{S}(r)$-$C(r\mathcal{E}_n)$, for every $r \in \mathbb{N}$.*

Let $\mathcal{S}(r)$-$E(\mathcal{E}_n)$ denote the set of $\mathcal{S}(r)$-Edgeworth equilibria for the economy $\mathcal{E}_n$. Note that $E(\mathcal{E}_n) \subset \mathcal{S}(r)$-$E(\mathcal{E}_n)$ for every set $\mathcal{S}(r)$ of admissible coalitions in each replica economy.

Given a set $\mathcal{S}$ of admissible coalitions in the economy $\mathcal{E}_n$, we remark that there is no natural way for defining the admissible coalition set in the replicated economies. For example, one might think that if the agent $i$ doesn't want to form a coalition with the agent $j$, then a copy of the agent $i$ will not form a coalition with a copy of the agent $j$. But one might also consider that the agent $i$ is willing to associate only a small fraction with the agent $j$. Alternatively, the agent $i$ forms a coalition with the agent $j$, but if she would be able to fractionize, then she would associate with $j$ no more than in a given fraction.

Following Aubin (1979) we define the pondered veto mechanism.

**Definition** 3.2 *An allocation $x$ is p-blocked by the coalition $S$ via the allocation $y$ if there exist $\alpha_s \in (0, 1]$, for each $s \in S$, such that $\sum_{s \in S} \alpha_s y_s = \sum_{s \in S} \alpha_s \omega_s$, and $y_s \succ_s x_s$, for every $s \in S$. The fuzzy core of an economy $\mathcal{E}_n$ , that we denote by $FC(\mathcal{E}_n)$ , is the set of all feasible allocations which cannot be p-blocked.*

This definition of fuzzy core is equivalent to the one due to Aubin (1979). However, it is important to remark that we require the coefficients $\alpha_i$ to be strictly positive for every agent forming the coalition. We will refer to this way of blocking as pondered veto mechanism.

Considering again a subset $\mathcal{S}$ of admissible coalitions, we introduce the concept of $\mathcal{S}$-fuzzy core.

**Definition** 3.3 *The $\mathcal{S}$-fuzzy core of an economy is the set of all feasible allocations which are not p-blocked by any coalition $S \in \mathcal{S}$.*

Let $\mathcal{S}$-$FC(\mathcal{E}_n)$ denote the $\mathcal{S}$-fuzzy core of the economy $\mathcal{E}_n$. Note that $FC(\mathcal{E}_n) \subset \mathcal{S}$-$FC(\mathcal{E}_n) \subset \mathcal{S}$-$C(\mathcal{E}_n)$, whatever subset $\mathcal{S}$ of admissible coalitions may be. Similar properties to those appointed for the $\mathcal{S}$-core can be obtained for the $\mathcal{S}$-fuzzy core.

Now, let us extend the restricted veto mechanism to the pondered veto mechanism. For this, we consider the restricted and pondered veto in a finite economy $\mathcal{E}$, in relation to the restricted veto in the replicated economies. So, given a coalition $S$ in $\mathcal{E}$, let $S(r)$ denote the set of coalitions in the economy $r\mathcal{E}$ consisting of $r_i$ agents of type $i$, with $0 < r_i \leq r$ if $i \in S$ and $r_i = 0$ if $i \notin S$. Let $\mathcal{S}(r) = \bigcup_{S \in \mathcal{S}} S(r)$. That is, $\mathcal{S}(r)$ is the set of coalitions in the $r$-fold replica economy, given by:

$$\mathcal{S}(r) = \left\{ S_r \subset rN | S_r = \bigcup_{\substack{i \in S \\ j \in J}} \{ij\} \text{ for some } S \in \mathcal{S} \text{ and } J \subset \{1, \ldots, r\} \right\}$$

Next we show that, if preferences are continuous, then the pondered veto is equivalent to the pondered veto with rational participation rates. That is, the $\mathcal{S}$-fuzzy core coincides with the $\mathcal{S}(r)$-Edgeworth equilibria. With this result we obtain a characterization of the $\mathcal{S}(r)$-Edgeworth equilibrium, without appealing to the sequence of replicated economies.

**Theorem** 3.1 *Let $\mathcal{E}$ be an economy with $n$ agents. Let an ordered Banach space $X$ be the commodity space. Suppose that preferences are continuous and convex. Then $\mathcal{S}$-$FC(\mathcal{E}) = \mathcal{S}(r)$-$E(\mathcal{E})$, whatever the set $\mathcal{S}$ of admissible coalitions in $\mathcal{E}$ may be.*

*Proof.* By convexity of preferences and by definition of the sets $\mathcal{S}(r)$, it is obviously verified that $\mathcal{S}$-$FC(\mathcal{E}) \subset \mathcal{S}(r)$-$E(\mathcal{E})$. To show the converse, suppose that the allocation $x = (x_1, \ldots, x_n) \notin \mathcal{S}$-$FC(\mathcal{E})$. Then, there exist $S \in \mathcal{S}$, $y_i \in X_+$ and $\alpha_i \in (0, 1]$, for every $i \in S$, such that $\sum_{i \in S} \alpha_i y_i = \sum_{i \in S} \alpha_i \omega_i$ and $y_i \succ_i x_i$ for all $i \in S$. For each $k \in \mathbb{N}$, let $\alpha_i^k = E[k\alpha_i + 1]$, where $E[t]$ denotes the entire part of the real number $t$. Let $y_i^k = \dfrac{k\alpha_i}{\alpha_i^k}(y_i - \omega_i) + \omega_i \in X_+$. Since $\lim_{k \to \infty} \dfrac{k\alpha_i}{\alpha_i^k} = 1$, then $\lim_{k \to \infty} y_i^k = y_i$. By continuity of preferences, there exists $k_0$ such that $y_i^k \succ_i y_i$ for every $i \in S$, and $k \geq k_0$. It is also verified that $\sum_{i \in S} \alpha_i^k y_i^k = \sum_{i \in S} k\alpha_i (y_i - \omega_i) + \alpha_i^k \omega_i = k \left( \sum_{i \in S} \alpha_i y_i - \sum_{i \in S} \alpha_i \omega_i \right) + \sum_{i \in S} \alpha_i^k \omega_i = \sum_{i \in S} \alpha_i^k \omega_i$. Therefore, the coalition with $\alpha_i^k$ agents of type $i$, for all $i \in S$, blocks the allocation $kx$ in the $k$-fold replica economy $k\mathcal{E}$, with $k \geq k_0$.

Q.E.D.

This result generalizes the equivalence between the set of Edgeworth equilibria and the fuzzy core, in the finite as well as infinite dimensional set up.

Now, consider a continuum economy $\mathcal{E}_c$, with $X$ as commodity space, in which only a finite number of different agents can be distinguished. The set of agents is represented by $I = [0,1] = \bigcup_{i=1}^{n} I_i$, where $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $I_n = \left[\frac{n-1}{n}, 1\right]$ denotes the set of agent of type $i$. Each consumer $t \in I_i$ is characterized by her consumption set $X_+$, her preference relation $\succeq_t = \succeq_i$ and her initial endowment $\omega(t) = \omega_i \in X_+$. Following García and Hervés (1993), this continuum economy $\mathcal{E}_c$ can be interpreted as an economy with $n$ agents, where the agent $i$ is the representative of infinite identical agents. For this, we associate to the economy $\mathcal{E}_c$ a discrete economy $\mathcal{E}_n$ with $n$ agents, where each agent $i \in N = \{1, \ldots, n\}$ is characterized by her preference relation $\succeq_i$ on $X_+$, and her initial endowments $\omega_i \in X_+$. Then, an allocation $f$ in $\mathcal{E}_c$ can be interpreted as an allocation $x = (x_1, \ldots, x_n)$ in $\mathcal{E}_n$, being $x_i = \dfrac{1}{\mu(I_i)} \displaystyle\int_{I_i} f(t) d\mu(t)$. Reciprocally, an allocation $x$ in $\mathcal{E}_n$ can be interpreted as an allocation $f$ in $\mathcal{E}_c$, where $f$ is the set function given by $f(t) = x_i$, if $t \in I_i$.

Given a set $\mathcal{S}_n$ of coalitions in $\mathcal{E}_n$ , we define the following set $\mathcal{S}_c$ of coalitions in $\mathcal{E}_c$ :

$$\mathcal{S}_c = \left\{ S_c \subset [0,1] \mid \text{there exists } \ S_n \in \mathcal{S}_n, \text{ such that } \mu(S_c \bigcap I_i) > 0 \text{ iff } i \in S_n \right\}$$

Reciprocally, given a set $\mathcal{S}_c$ of coalitions in $\mathcal{E}_c$ , we define the following set $\mathcal{S}_n$ of coalitions in $\mathcal{E}_n$ :

$$\mathcal{S}_n = \left\{ S_n \subset \{1, \ldots, n\} \mid \text{there exists } S_c \in \mathcal{S}_c, \text{ such that } i \in S_n \text{ iff } \mu(S_c \bigcap I_i) > 0 \right\}$$

Next we show the equivalence between the $\mathcal{S}_n$-fuzzy core of $\mathcal{E}_n$ and the $\mathcal{S}_c$-core of $\mathcal{E}_c$ , addressing economies with a finite number of commodities as well as economies with an infinite dimensional commodity space.

**Theorem** 3.2 *Let $\mathcal{E}_n$ be an economy with $n$ agents. The commodity space $X$ is an ordered Banach space. Let $\mathcal{E}_c$ be the associated continuum economy with $n$ types of agents. Suppose that preferences are continuous and convex on $X_+$. Then, the the following statements hold:*

*If the allocation $f \in \mathcal{S}_c$-$C(\mathcal{E}_c)$, then $x = (x_1, \ldots, x_n) \in \mathcal{S}_n$-$FC(\mathcal{E}_n)$, where $x_i = \dfrac{1}{\mu(I_i)} \displaystyle\int_{I_i} f(t) d\mu(t)$.*
*Reciprocally, if $x = (x_1, \ldots, x_n) \in \mathcal{S}_n$-$FC(\mathcal{E}_n)$, then $f \in \mathcal{S}_c$-$C(\mathcal{E}_c)$, where $f(t) = x_i$ if $t \in I_i$.*

*Proof.* Suppose that $x$, given by $x_i = \dfrac{1}{\mu(I_i)} \displaystyle\int_{I_i} f(t) d\mu(t)$, is $p$-blocked by the coalition $S_n \in \mathcal{S}_n$ via $(y_i)_{i \in S_n}$. Then, there exist $\alpha_i$, with $0 < \alpha_i \leq \frac{1}{n}$, such that $\displaystyle\sum_{i \in S_n} \alpha_i y_i = \sum_{i \in S_n} \alpha_i \omega_i$ and $y_i \succ_i x_i$, for all $i \in S_n$. Thus, for each $i \in S_n$ there exists $T_i \subset I_i$, with $\mu(T_i) > 0$, such that $y_i \succ f(t)$, for all $t \in T_i \subset I_i$ (see the lemma in García and Hervés (1993)). Let $\beta = \min_{i \in S_n}\{\mu(T_i)\}$. and let $S_i \subset T_i$, with $\mu(S_i) = \beta\alpha_i$, for each $i \in S_n$. Consider $S_c = \bigcup_{i \in S_n} S_i$ and define $y(t) = y_i$, if $t \in S_i$. It is verified that $\int_{S_c} y(t)d\mu(t) = \sum_{i \in S_n} \beta\alpha_i y_i = \sum_{i \in S_n} \beta\alpha_i \omega_i = \int_{S_c} \omega(t)d\mu(t)$ and $y_i \succ_i f(t)$, for all $t \in S_i$, and every $i \in S_n$. Therefore, the coalition $S_c \in \mathcal{S}_c$ blocks $f$ via $y$.

Reciprocally, suppose that $f$, given by $f(t) = x_i$, if $t \in I_i$, is blocked by $S_c \in \mathcal{S}_c$. Let $S_n = \{i \in \{1, \ldots, n\} | \mu(S_c \bigcap I_i) > 0\}$. By convexity and continuity of preferences, there exists $y_i \in X_+$, for each $i \in S_n$, such that $\sum_{i \in S_n} \mu(S_n \bigcap I_n)\, y_i = \sum_{i \in S_n} \mu(S_n \bigcap I_n)\, \omega_i$ and $y_i \succ_t f(t)$, for all $t \in S_c \bigcap I_i$. Therefore, $x$ is $p$-blocked by the coalition $S_n$.

Q.E.D.

Hüsseinov (1994) considers economies with a finite dimensional commodity space and, under convexity of preferences, he shows that $x \in FC(\mathcal{E}_n)$ iff $f \in C(\mathcal{E}_c)$, being $f(t) = x_i$ if $t \in I_i$. In the same paper, assuming also continuity and monotonicity on preferences, it is shown that if $f \in C(\mathcal{E}_c)$, then $x = (x_1, \ldots, x_n) \in FC(\mathcal{E}_n)$, with $x_i = \frac{1}{\mu(I_i)} \int_{I_i} f(t)d\mu(t)$. Note that both results are particular cases of theorem 3.2.

We remark that in theorem 3.1 and theorem 3.2 it is sufficient to assume that the consumption sets are convex instead of supposing that they correspond to the positive orthant. [1]

Next we obtain our main result.

**Theorem** 3.3 *Let $\mathcal{E}_n$ be an economy with $n$ agents and $\ell$ commodities. Assume that preferences are continuous, monotone and convex. Then, $FC(\mathcal{E}_n) = \mathcal{S}$-$FC(\mathcal{E}_n)$ , for every $\mathcal{S}$, such that the coalition of all agents $N$ belongs to $\mathcal{S}$.*

*Proof.* Obviously $FC(\mathcal{E}_n) \subset \mathcal{S}$-$FC(\mathcal{E}_n)$ , whatever $\mathcal{S}$ may be. Suppose that $x = (x_1, \ldots, x_n) \notin FC(\mathcal{E}_n)$ . Then, there exists a coalition $S \subset N$ which $p$-blocks $x$ via $(y_i)_{i \in S}$. That is, there exist $\alpha_i$, with $0 < \alpha_i \leq \frac{1}{n}$, such that $\sum_{i \in S} \alpha_i y_i = \sum_{i \in S} \alpha_i \omega_i$ and $y_i \succ_i x_i$, for every $i \in S$. Let $S_c \subset [0, 1]$ be a coalition in the continuum economy $\mathcal{E}_c$ , such that $\mu(S_c \cap I_i) = \alpha_i$. Let $y(t) = y_i$ if $t \in S_c \cap I_i$ and let $f(t) = x_i$ if $t \in I_i$. Thus, the coalition $S_c$ blocks the allocation $f$ in $\mathcal{E}_c$ by via $y$. By Vind's (1972) result, there exist $S'_c \subset [0, 1]$, with $\mu(S'_c) > \frac{n-1}{n}$, such that $S'_c$ blocks $f$. The convexity of preferences implies that there exist $(y'_i)_{i=1}^n$, such that $y'_i \succ_i x_i$ for every $i \in \{1, \ldots, n\}$ and $\sum_{i=1}^{n} \mu(S'_c \cap I_i)\, y'_i = \sum_{i=1}^{n} \mu(S'_c \cap I_i)\, \omega_i$. Therefore, the

[1]This remark was pointed out by an anonymous referee.

coalition $N$ $p$-blocks $x$ via $y'$.

Q.E.D.

In this way, we conclude that, in order to get the fuzzy core, it is enough to consider the pondered veto mechanism restricted only to one coalition, namely, the coalition of all agents. Then, using theorem 3.1, we obtain that the pondered veto mechanism, restricted to the coalition formed by all agents, results on the set of Edgeworth equilibria.

Let $W(\mathcal{E})$ denote the set of Walrasian allocations for the economy $\mathcal{E}$. Aubin (1979) shows that if $\mathcal{E}_n$ is an economy, under the assumptions in theorem 4.3 and with strictly positive total endowments, then $FC(\mathcal{E}_n) = W(\mathcal{E}_n)$. We show that the pondered veto power of the coalition formed by all agents, eliminates the non Walrasian allocations.

**Corollary** 3.1 *Let $\mathcal{E}_n$ be an economy with $n$ agents and $\ell$ commodities. Assume that preferences are continuous, convex and monotone. Assume also that the total resources vector $\omega = \sum_{i=1}^{n} \omega_i$ is strictly positive. Let $\mathcal{S}$ any set of coalitions containing the coalition $N$ of all agents. Then $\mathcal{S}$-$FC(\mathcal{E}_n) = W(\mathcal{E}_n)$.*

We notice that the equivalence between the fuzzy core and the set of Walrasian allocations does still hold for economies with an infinite dimensional commodity space. García and Hervés (1993), under the hypothesis of properness of preferences, show the core-Walras equivalence for continuum economies with $n$ types of agents and with an ordered Banach space as commodity space. The requeriments which guarantee the core-Walras equivalence and theorem 3.2 allow us to conclude that the pondered veto mechanism coincides with the Walrasian mechanism for economies with a finite number of agents and with an infinite dimensional commodity space.

In this section, the convexity of preferences has been required in order to obtain that the pondered veto mechanism, restricted to the coalition of all agents, results on the set of Walrasian allocations. It is known that, without convexity of preferences, the pondered veto mechanism does not coincide with the Walrasian mechanism. That is why Hüsseinov (1994) enlarges the set of coalitions, considering which he called fuzzy coalitions. However, the veto of such fuzzy coalitions is precisely the the pondered veto in the replicated economies. In fact, even for economies with non convex preferences, the pondered veto mechanism in every replicated economies $r\mathcal{E}_n$ eliminates the non Walrasian allocations. Moreover, Caratheodory's theorem allows us to conclude that it is enough to consider the pondered veto mechanism until the $(\ell + 1)$-fold replica economy, in order to get the set of Edgeworth equilibria or, alternatively, the set of Walrasian allocations for the economy $\mathcal{E}_n$.

# 4 Final Remarks

Florenzano (1990) shows the existence of Walrasian equilibrium, fuzzy core and Edgeworth equilibrium of a production economy without ordered preferences. Both the restricted and pondered veto mechanism may be analyze within the non ordered preference set up.

The restricted veto mechanism provides a framework where we have formulated known results. This is the case of the results by Schmeidler (1972), Grodal (1972) and Vind (1972), as we have remarked in section 2. We observe that Hansen's (1969) result can also be formulated within this framework. Recently Gilles, Haller and Ruys (1998) introduce the notion of semi-core which imposes a restriction on the set of "admissible" coalitions and they show that this restriction does not affect the fundamental equivalence property. Thus, their result can be included within the same restricted veto mechanism framework in terms of $\mathcal{S}$-cores.

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