SYNCHRONISATION CONDITIONS IN ASYNCHRONOUS WIND PARKS

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ABSTRACT: the purpose of this work is to study the conditions under which synchronisation is possible in asynchronous wind parks. Synchronisation is defined as the situation where blades of different wind turbines pass simultaneously in front of their towers. Tower shadow effect is generally considered responsible for part of power quality problems originated by wind turbines. The way of summing the effects of several wind turbines is important. When synchronisation of several wind turbines occurs, the addition effects can be reinforced.
Keywords: Dynamic Models, Power Quality, Synchronisation of Asynchronous Wind Turbines, Wind Farm.

1 INTRODUCTION

When several wind turbines rotate at the same speed and their blades pass in front of their towers at the same time, it can be due to a process of synchronisation. In this situation, tower shadow effects of all the wind turbines can be added, which can have a significant effect on the electrical system.

Interest in studying synchronism and the synchronisation process of wind turbines is due to the way power from these machines oscillates due to rotational sampling effect (tower shadow) with an amplitude of about 20 % of the mean power value, according to some authors and measurements [1, 2, 3]. The final effects of this are voltage variations and possible flicker.

Generally, the square root of n rule is accepted to add the effects of different wind turbines on the electrical network [1, 4, 5], where n is the number of wind turbines in the wind park, assuming they are identical. Some authors have taken measurements which agree with this idea [4], but some others hold that they have observed a trend to synchronisation in groups of wind turbines, and consequently the total effect is higher [1].

Under certain conditions, synchronisation of wind turbines can be simulated [6]. These conditions mean they have to be identical, and the mean value of the mechanical power must be the same for all them.

Synchronisation, as shown in Figs. 1.a and 1.b, means that the difference of mechanical angles tend to zero and mechanical and electrical powers, consequently, tend to the same values. It is a very slow process, which can be seen in the mentioned figures. At the same time, rapid variations caused by mechanical power fluctuations occur, as can be seen in Fig. 2.

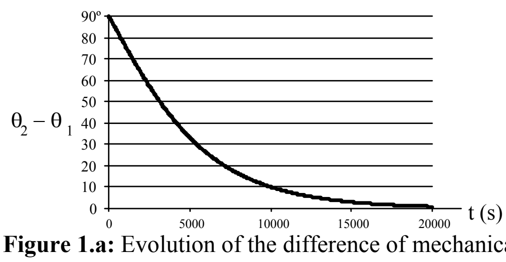

Figure 1.a: Evolution of the difference of mechanical blade angles of two identical wind turbines

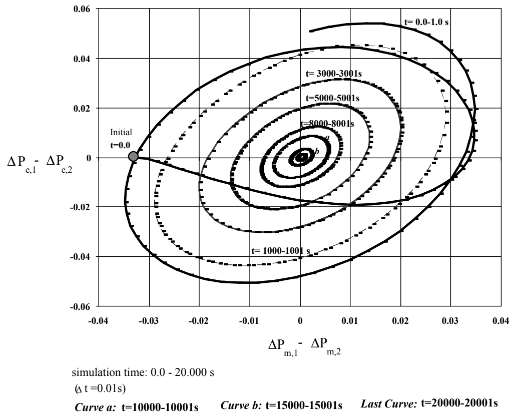

Figure 1.b: Evolution of electrical and mechanical powers for the synchronisation process

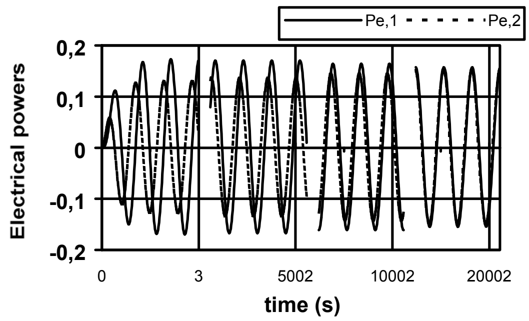

Figure 2: Evolution of electrical powers

Some explanations have been given for synchronisation of asynchronous wind turbines in a wind park assuming steady-state models [7]. The purpose of this paper is to give a complete explanation of synchronism conditions in wind turbines connected to the same point of common connection, by using several models based on the one proposed in [8, 9].

The analysis of synchronism and non-synchronism conditions is done by sequentially using two dynamic models of a wind park: linear dynamic model and dynamic model in incremental notation. The linear dynamic model is applied to mechanical sinusoidal fluctuations ΔP_m produced by rotational sampling effects, and has a lower time constant (a few seconds). The results of simulation with the linear model are used to calculate variations in constant values of electrical and mechanical powers $\delta P_m, \delta P_e$. These powers are applied to dynamic models and modifications of rotational and mechanical angles are obtained that can produce synchronism of wind turbines, depending on wind turbine parameters and conditions. For this second simulation the time constant is higher (thousand seconds).

2 MODELLING OF INDUCTION GENERATORS AND ASYNCHRONOUS WIND PARKS

The induction generator can be dynamically modelled as a voltage source $\underline{E}'$ behind the impedance R+j·X′, as can be consulted in [8, 9]. This was the first model used for simulations. As the model can be read in the given references, a description of it is not a purpose of this paper.

Two models are described here, which are the so called incremental dynamic model and the linear dynamic one.

2.1 Incremental dynamic model of an asynchronous wind turbine

Based on the mentioned dynamic model, if the induction generator is assumed to be in an initial steady-state situation described by a set of values $\underline{E}'_0$ (internal voltage), s_0 (slip), $\underline{I}_0$ (stator current), P_0 (mechanical power), $\underline{U}_0$ (terminal voltage), the value of $\Delta\underline{E}'$ can be calculated by integrating the following equation:

$$\frac{d\Delta\underline{E}'}{dt} = -\underline{z}\bullet\Delta\underline{E}' - j\bullet\omega_s\bullet\Delta s\bullet\underline{E}_0' + \underline{z}'\bullet\Delta\underline{U} - j\bullet\omega_s\bullet\Delta s\bullet\Delta\underline{E}'$$

where:

$$\underline{z} = j\bullet\omega_s\bullet s_0 + \frac{1}{T'_0}\bullet\left(1 + \frac{j\bullet(X_0 - X')}{R_s + j\bullet X'}\right)$$

$$\underline{z}' = \frac{1}{T'_0}\bullet\frac{j\bullet(X_0 - X')}{R_s + j\bullet X'}$$

and T'_0, X_0 and X' are obtained from the generator parameters.

As for electromechanical equations, the following can be written:

$$\Delta P_m - \Delta P_e = 2\bullet H\bullet(1 - s_0 - \Delta s)\bullet\frac{d\Delta s}{dt}$$

$$\frac{d\Delta\theta}{dt} = -\Omega_s\bullet\Delta s$$

The electrical power, ΔP_e, is calculated by:

$$\Delta P_e = \mathbf{Re}\left\{\begin{matrix} -\underline{y}\bullet\Delta\underline{E}'^* + \underline{I}^*_{s,0}\bullet\Delta\underline{E}' + \underline{y}\bullet\Delta\underline{U}^* \\ -\underline{y}'\bullet\Delta\underline{E}'\bullet\Delta\underline{E}'^* + \underline{y}'\bullet\Delta\underline{E}'\bullet\Delta\underline{U}^* \end{matrix}\right\}$$

where:

$$\underline{y} = \frac{\underline{E}_0^*}{R_s - j\bullet X'}$$

$$\underline{y}' = \frac{1}{R_s - j\bullet X'}$$

and the mechanical power is:

$$\Delta P_m(t) = P_s\bullet\sin(\theta_0 + \Omega_0\bullet t + \Delta\theta)$$

where $\Omega_0 = (1 - s_0)\bullet\Omega_s$.

2.2 Linear dynamic model

Assuming small changes in the last equations, $\Delta\underline{E}'\bullet\Delta s \approx 0$, $\Delta s << s_0$, $\Delta\underline{E}'\bullet\Delta\underline{E}'^* \approx 0$, $\Delta\underline{E}'\bullet\Delta\underline{U}^* \approx 0$ and $\sin(\Delta\theta) = \Delta\theta \approx 0$, a linear differential first order system results [10], defined by:

$$\dot{\mathbf{x}} = \mathbf{A}\bullet\mathbf{x} + \mathbf{c}\bullet\Delta U + \mathbf{b}\bullet\Delta P'_m$$

2.3 Dynamic model of a wind park

Applying the dynamic model to a wind park with n wind turbines, from the nodal analysis of the electrical circuit, the following equation can be written:

$$\Delta\underline{\mathbf{U}} = \underline{\mathbf{K}}\bullet\Delta\underline{\mathbf{E}'}$$

The electrical equations of the wind turbines are expressed as:

$$\frac{d}{dt}\left[\Delta\underline{E}_i'\right] = \underline{\mathbf{B}}\bullet\Delta\underline{\mathbf{E}'} + \underline{\mathbf{C}}\bullet\Delta\,\mathbf{s} - j\bullet\omega_s\bullet\left[\Delta s_i\bullet\Delta\underline{E}_i'\right]$$

which in rectangular components are:

$$\frac{d}{dt}\begin{bmatrix}\Delta E'^r_i \\ \Delta E'^m_i\end{bmatrix} = \mathbf{B'}\bullet\Delta\,\mathbf{E'} + \mathbf{C'}\bullet\Delta\,\mathbf{s} + \omega_s\bullet\begin{bmatrix}\Delta s_i\bullet\Delta E'^m_i \\ -\Delta s_i\bullet\Delta E'^r_i\end{bmatrix}$$

where $\Delta\,\mathbf{E'}$ is the vector of real and imaginary components of the complex vector $\Delta\underline{\mathbf{E}'}$, and **B**' and **C**' are real matrices.

The electromechanical equations are:

$$\Delta P_{m,i} - \Delta P_{e,i} = h_i\bullet\frac{d\Delta s_i}{dt}$$

$$\frac{d\Delta\theta_i}{dt} = -\Omega_s\bullet\Delta s_i$$

where $h_i = 2\bullet H_i\bullet(1 - s_i)$.

The electrical power, $\Delta P_{e,i}$, can be calculated as:

$$\left[\Delta P_{e,i}\right] = \mathbf{M}\bullet\Delta\,\mathbf{E'} + \mathbf{Re}\left\{-\left[\underline{y}_i'\bullet\Delta\underline{E}'_i\bullet\Delta\underline{E}'^*_i\right] + \left[\Delta\underline{E}'_i\bullet\underline{\mathbf{N}}\bullet\Delta\underline{\mathbf{E}'}\right]\right\}$$

where **M** is a real matrix.

And the mechanical power is:

$$\Delta P_{m,i} = P_{s,i}\bullet\sin(\theta_{0,i} + \Omega_{0,i}\bullet t + \Delta\theta_i)$$

where $\Omega_{0,i} = (1 - s_{0,i})\bullet\Omega_s$.

2.4 Mechanical fluctuations

Assuming small changes: $\Delta s_i\bullet\Delta\underline{E}'_i \approx 0$, $\Delta\underline{E}'_i\bullet\Delta\underline{E}'^*_j \approx 0$ and $\sin(\Delta\theta_i) = \Delta\theta_i \approx 0$ for (i, j =1, ..., n), the following equation can be written, by means of the Fourier Transform:

$$\underline{X} = \underline{D} \bullet \Delta \underline{p}$$

where:

$$D = \left(j \bullet \Omega \bullet 1 - A\right)^{-1} \bullet H$$

Ω is the imaginary variable in the Fourier analysis.
1 is the unitary matrix.
H is the diagonal matrix of $1/h_i$ components.
A is the real matrix with the relations between the variables $\Delta \underline{E}'_i$, Δs_i and $\Delta P_{e,i}$.
$\Delta\,\mathbf{p}$ is the complex vector of $\Delta P_{m,i}$ values.

$\Delta\,\mathbf{E}'$, $\Delta\,\mathbf{s}$ and $\Delta\boldsymbol{\theta}$ can be defined by the expression:

$$\Delta E_i^r(t) = X_i^r \bullet \sin(\Omega_0 \bullet t) + X_i^m \bullet \mathbf{cos}(\Omega_0 \bullet t)$$
$$\Delta E_i^m(t) = X_{(i+1)}^r \bullet \sin(\Omega_0 \bullet t) + X_{(i+1)}^m \bullet \mathbf{cos}(\Omega_0 \bullet t)$$
$$\Delta s_i(t) = X_{(i+2)}^r \bullet \sin(\Omega_0 \bullet t) + X_{(i+2)}^m \bullet \mathbf{cos}(\Omega_0 \bullet t)$$
$$\Delta\theta_i(t) = -\left(\frac{\Omega_s}{\Omega_0}\right)\left(X_{(i+2)}^m \bullet \sin(\Omega_0 \bullet t) - X_{(i+2)}^r \bullet \mathbf{cos}(\Omega_0 \bullet t)\right)$$

where all variables have the same rotational speed (frequency) Ω_0. This situation does not happen when the working points of the wind turbines are different: $P_{m,1,0}, P_{m,2,0}, \cdots, P_{m,n,0}$; $s_{1,0}, s_{2,0}, \cdots, s_{n,0}$. In this case the last equations have a multi-frequency expression, such as:

$$\Delta E_i^r(t) = \sum_{k=1}^{n} X^{(k)r}_i \bullet \sin(\Omega_{0,k} \bullet t) + \sum_{k=1}^{n} X^{(k)m}_i \bullet \mathbf{cos}(\Omega_{0,k} \bullet t)$$

If small changes are now considered, the following expression is valid:

$$\begin{bmatrix} \delta E'^r_i \\ \delta E'^m_i \end{bmatrix} \equiv \omega_s \bullet \begin{bmatrix} \Delta s_i(t) \bullet \Delta E'^m_i(t) \\ -\Delta s_i(t) \bullet \Delta E'^r_i(t) \end{bmatrix}$$
$$\left[\delta P_{e,i}\right] = \mathbf{M} \bullet \delta\,\mathbf{E} \;+$$
$$\mathbf{Re}\left\{-\left[\underline{y}'_i \bullet \Delta\underline{E}'_i(t) \bullet \Delta\underline{E}'^*_i(t)\right] + \left[\Delta\underline{E}'_i(t) \bullet \sum_{k=1}^{m} \underline{y}'_k \bullet \Delta\underline{E}'_k(t)\right]\right\}$$
$$\delta P_{m,i} = P_{s,i} \bullet \sin(\theta_{0,i} + \Omega_{0,i} \bullet t + \Delta\theta_i(t)) - P_{s,i} \bullet \sin(\theta_{0,i} + \Omega_{0,i} \bullet t)$$
$$\approx P_{s,i} \bullet \mathbf{cos}(\theta_{0,i} + \Omega_{0,i} \bullet t) \bullet \Delta\theta_i(t)$$

With $\delta P_{m,i}$ and $\delta P_{e,i}$ the following equations are obtained:

$$\delta P_{m,i} - \delta P_{e,i} - \delta P'_{e,i} = h_i \bullet \frac{d\delta s_i}{dt}, \quad \frac{d\delta\theta}{dt} = -\Omega_s \bullet \delta s$$

where $\delta P'_{e,i}$ is an additional electrical power produced by slip variations δs_i, and can be defined by the following equation:

$$\frac{d}{dt}\left[\delta\,\mathbf{E}''\right] \equiv 0 = \mathbf{B}' \bullet \delta\,\mathbf{E}'' + \mathbf{C}' \bullet \delta\,\mathbf{s}$$
$$\left[\delta P'_{e,i}\right] = -\,\mathbf{M} \bullet \left(\mathbf{B}'\right)^{-1} \bullet \mathbf{C}' \bullet \delta\,\mathbf{s}$$

3 SYNCHRONISATION

The synchronisation analysis is defined for a wind park with identical wind turbines and identical initial conditions. In the other cases, powers do not tend to equal values and, consequently, the synchronism phenomenon is not possible.

The analysis of synchronism can be defined using one of the wind turbines as a reference. So, let us assume that machine 1 is taken as a reference:

$$(\delta P_{m,i} - \delta P_{m,1}) - (\delta P_{e,i} - \delta P_{e,1}) - (\delta P'_{e,i} - \delta P'_{e,1}) = h \bullet \frac{d\delta s_{i,1}}{dt}$$
$$\frac{d\delta\theta_{i,1}}{dt} \equiv \frac{d(\delta\theta_i - \delta\theta_1)}{dt} = -\Omega_s \bullet \delta s_{i,1}$$

where:

$$\delta P_{m,i} - \delta P_{m,1} = K'^m_{i,1} \bullet (P_{s,i}^2 - P_{s,1}^2) + K^m_{i,1} \bullet P_{s,i} \bullet P_{s,1} \bullet \sin(\theta_i - \theta_1)$$
$$K'^m_{i,1} = \frac{\Omega_s}{\Omega_0} \bullet D^r_{i+2,i+2}$$
$$K^m_{i,1} = \frac{\Omega_s}{\Omega_0} \bullet D^r_{i+2,1+2}$$
$$\delta P_{e,i} - \delta P_{e,1} = K'^e_{i,1} \bullet (P_{s,i}^2 - P_{s,1}^2) + K^e_{i,1} \bullet P_{s,i} \bullet P_{s,1} \bullet \sin(\theta_i - \theta_1)$$
$$K'^e_{i,1} = M_{i+2,i} \bullet (\Lambda^{xx}_{i,i,i} - \Lambda^{xx}_{i,1,1}) + M_{i+2,i+1} \bullet (\Lambda'^{xx}_{i,i,i} - \Lambda'^{xx}_{i,1,1})$$
$$K^e_{i,1} = M_{i+2,i} \bullet \Lambda^{xy}_{i,i,1} + M_{i+2,i+1} \bullet \Lambda'^{xy}_{i,i,1}$$
$$\delta P'_{e,i} - \delta P'_{e,1} = D^e_{i,1} \bullet (\delta s_i - \delta s_1)$$

These equations mean the following powers exist in a wind park:

- A constant power, $K'^{m,e}_{i,j} \bullet (P_{s,i}^2 - P_{s,j}^2)$, produced by different mechanical power fluctuation levels.
- A synchronisation power, $K^{m,e}_{i,j} \bullet \sin(\theta_i - \theta_j)$, function of mechanical power fluctuation levels and difference between mechanical angles.
- A damping power, $D^e_{i,j} \bullet (\delta s_i - \delta s_j)$.

The constant ($K'^{m,e}_{i,j}$), synchronisation ($K^{m,e}_{i,j}$) and damping ($D^e_{i,j}$) coefficients are expressed by the linear dynamic model in steady-state situation, and they are a function of parameters and the initial working point of wind turbines.

In an example with two identical wind turbines and $P_{s,1} = P_{s,2} = 0.1$, $h = 6.093$ and $\Omega_s = 10.62$, the dynamic equations are:

$$K \bullet \sin(\theta_0 + \delta\theta) - D \bullet (\delta s) = h \bullet \frac{d\delta s}{dt}$$
$$\frac{d\delta\theta}{dt} = -\Omega_s \bullet \delta s$$

where:

$$\theta_0 = \theta_{2,0} - \theta_{1,0}$$
$$\delta\theta = \delta\theta_2 - \delta\theta_1$$
$$\delta s = \delta s_2 - \delta s_1$$
$$K = 8.55 \bullet 10^{-3}$$
$$D = 121.42$$

Assuming $\sin(\delta\theta) \approx \delta\theta$ and $\cos(\delta\theta) \approx 1$, the last equations can be expressed as:

$$M \bullet \frac{d^2}{dt^2}(\delta\theta) = A + B \bullet \sin(\delta\theta) + C \bullet \frac{d(\delta\theta)}{dt}$$

where:

$$A = K \bullet \sin(\theta_0)\,,\; B = K \bullet \mathbf{cos}(\theta_0)$$
$$C = \frac{D}{\Omega_s}$$
$$M = -\frac{h}{\Omega_s}$$

The equation roots p_1 and p_2 of the differential equation can be expressed as:

$$p_1 = \frac{C}{2 \bullet M} \bullet \left(1 + \sqrt{1+\zeta}\right) \approx \frac{C}{M} = -19.92$$
$$p_2 = \frac{C}{2 \bullet M} \bullet \left(1 - \sqrt{1+\zeta}\right) \approx -\frac{K}{C} << p_1$$

where

$$\zeta = \frac{4 \bullet B \bullet M}{C^2} = -1.5 \bullet 10^{-4} \bullet \mathbf{cos}(\theta_0) << 1$$

So the variable $\theta = \theta_0 + \delta\theta$, for $\theta_0 \approx 10^{\circ}$ and $p_2 = -7.4 \bullet 10^{-4}$, can be defined as: $\theta = \theta_0 \bullet e^{p_2 \bullet t}$, whose evolution is similar to that given in Fig. 1.a obtained by simulation.

4 CONCLUSIONS

In order to study the synchronisation of wind turbines, the following assumptions were made:

- Two models of asynchronous generators were used: a linear dynamic model and an incremental one.
- Mechanical power fluctuations are considered to be sinusoidal.

Several conclusions can be extracted:

- Synchronisation is a very slow process.
- Synchronisation can be studied by means of second order differential equations.
- Synchronisation happens when the wind turbines are identical.
- For different wind parks a synchronisation process seems to be very improbable.

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