



Geometry Expressions: A Constraint Based Interactive Geometry System

Philip Todd
Saltire Software
Beaverton, OR
philt@saltire.com



Outline

- Project History (1987 – 2006)
- Architecture of an Interactive Symbolic Geometry System
- Examples of Use



Project History

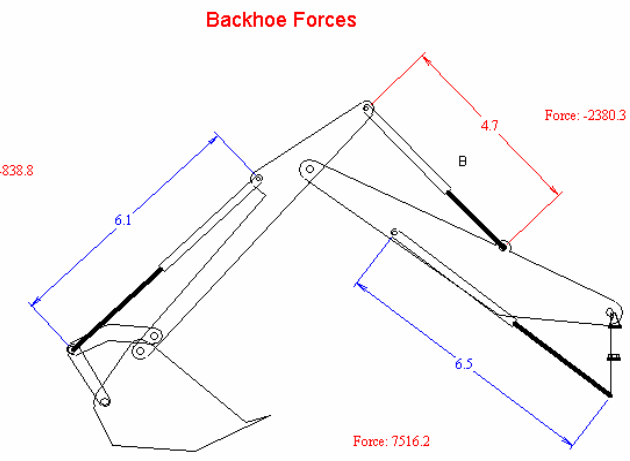
- 1987 Symbolic Geometry project at Tektronix Computer Research Lab
- 1989 Saltire Software founded
- 1992 – 1995 NSF SBIR research grant
- 2003 demo old prototype on new computer
- May 2006 Geometry Expressions released

Analytix 1989-1996

Analytix-C:\PRODUCTS\AX\ANALYTIX\SAMPLES\BACKHOE1.AX

File Edit Sketch Dimension Constrain View Defaults Tools Attributes Analysis Help

Consistently Dimensioned



Backhoe Forces

Force: -838.8

6.1

Force: -2380.3

4.7

B

Force: 7516.2

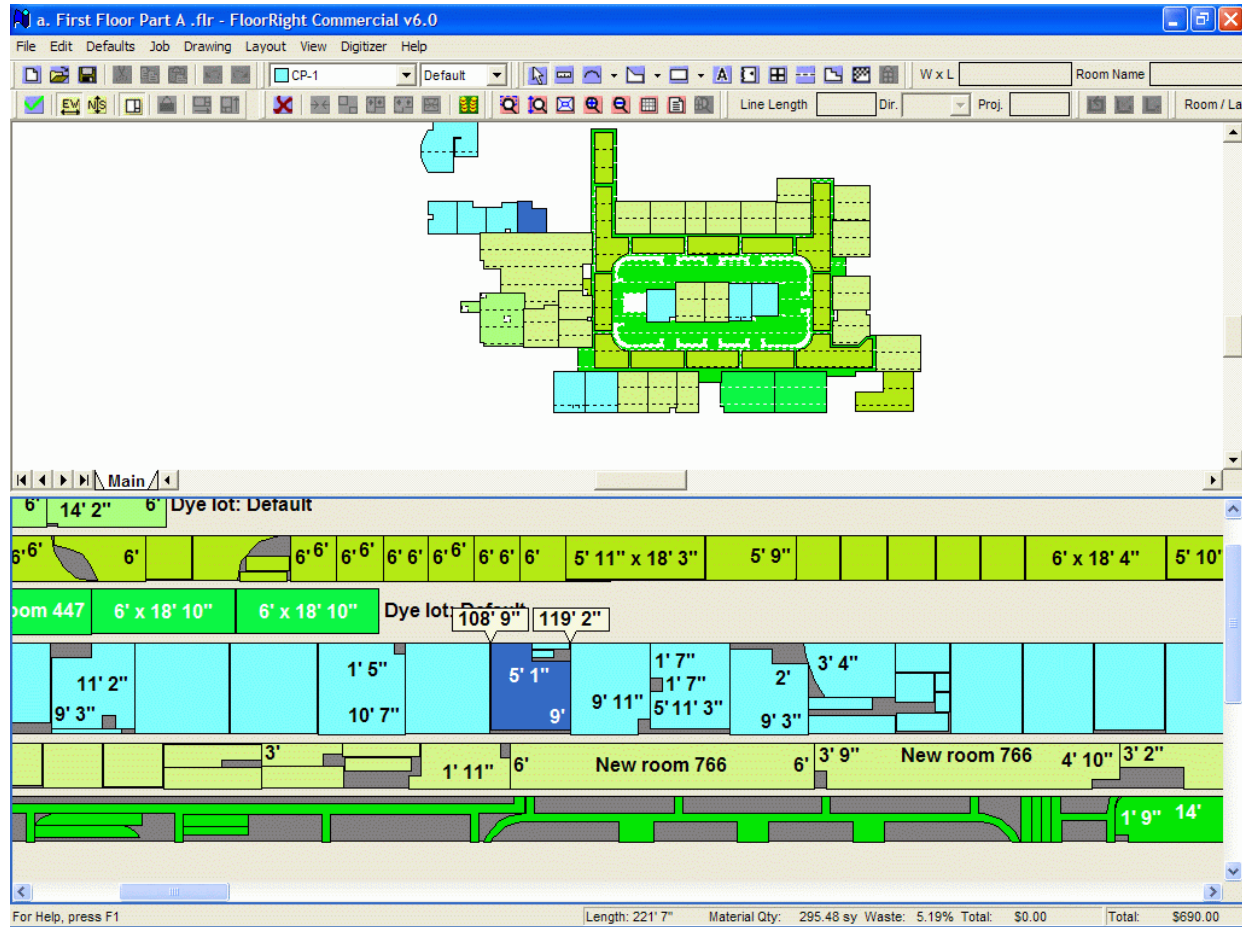
6.5

To move the backhoe and see the forces update:
 Double-click on actuator B's dimension & change its value (e.g. change the 4.7 to 4)

To do the same thing with a slider control:
 Use the Tools | Increment... menu, click on dimension B, then
 set the Initial Value, Step Size, and Final Value appropriately (e.g. 3, 0.2, and 6 respectively)



FloorRight 1996-2006



Calculators 1996-2006





Architecture

- Archetypal Variable Value Manager
- Constraint graph algorithms
- Algebra system
- MathML

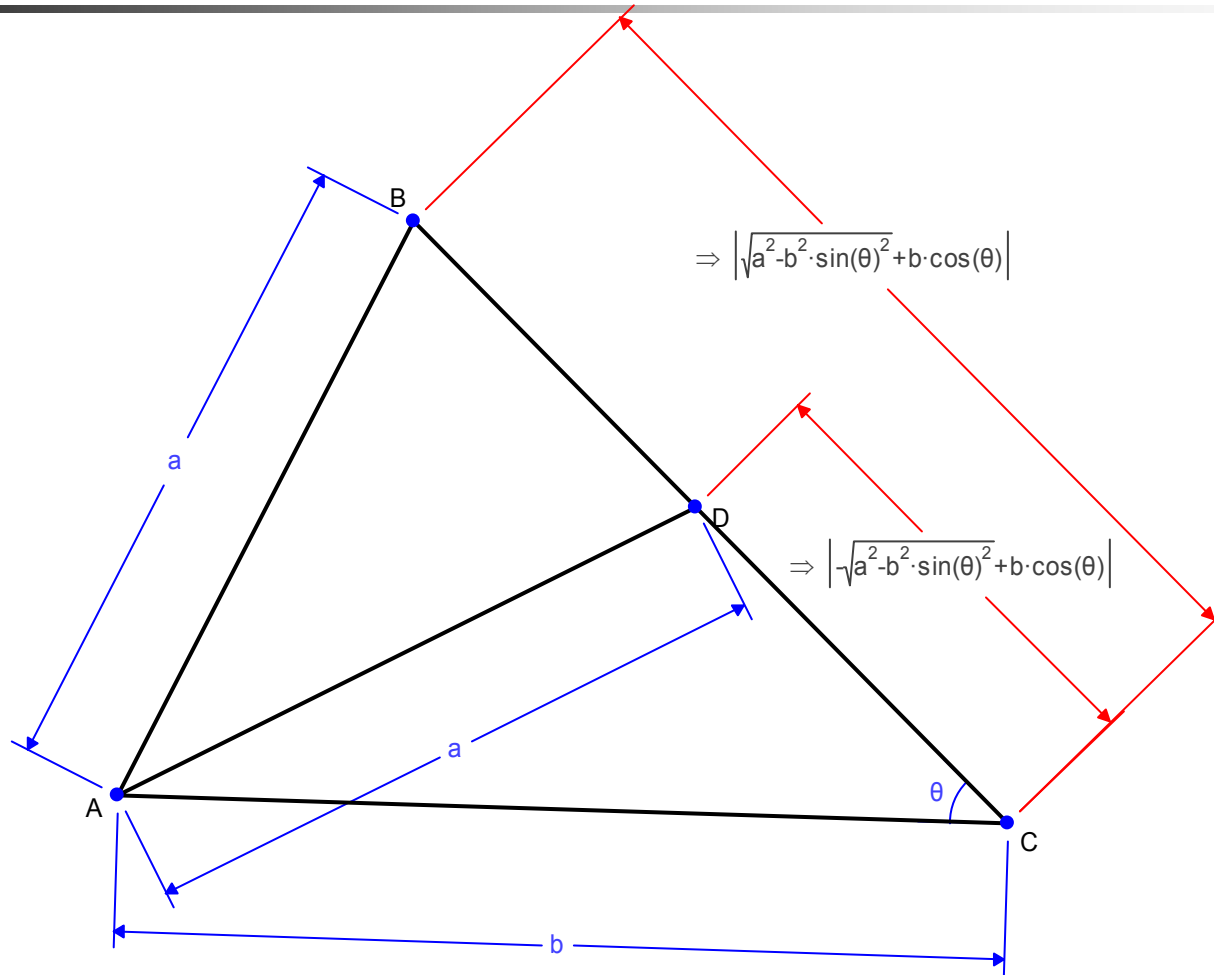


Archetypal Variable Value Manager

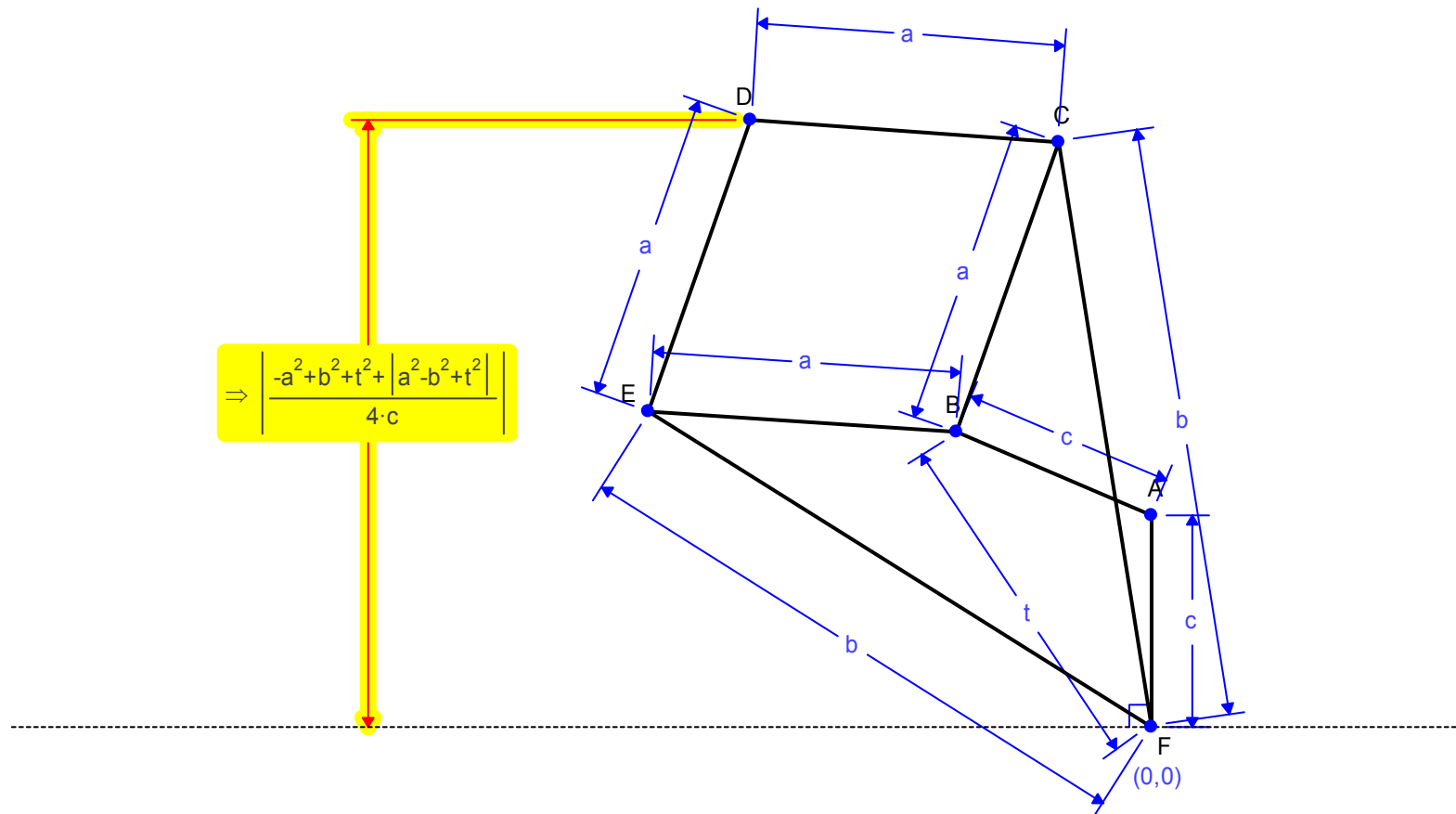
Uses for archetypal real values

- Drawing
- Construction disambiguation
- Automatic assumption mechanism

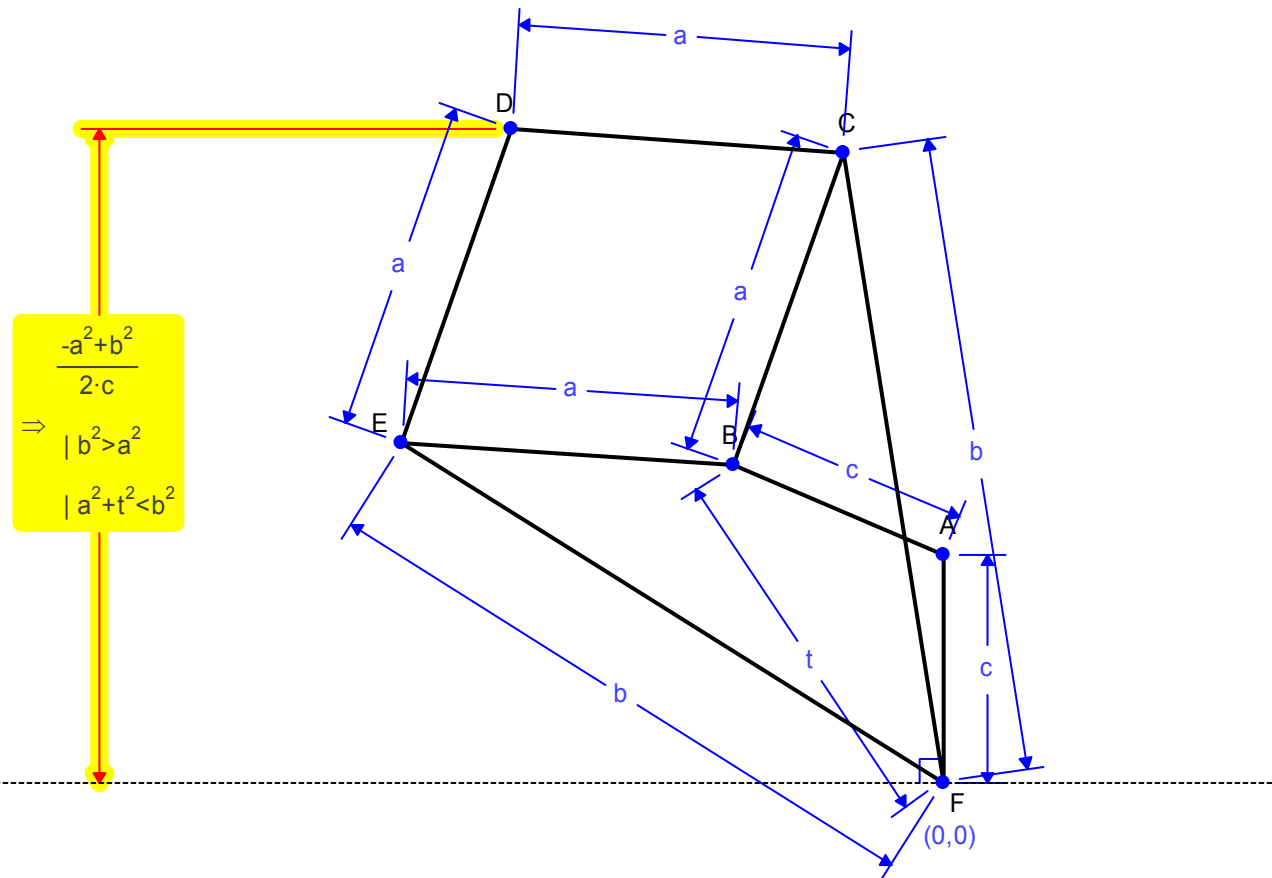
Construction Disambiguation



Automatic Assumption Mechanism



Automatic Assumption Mechanism



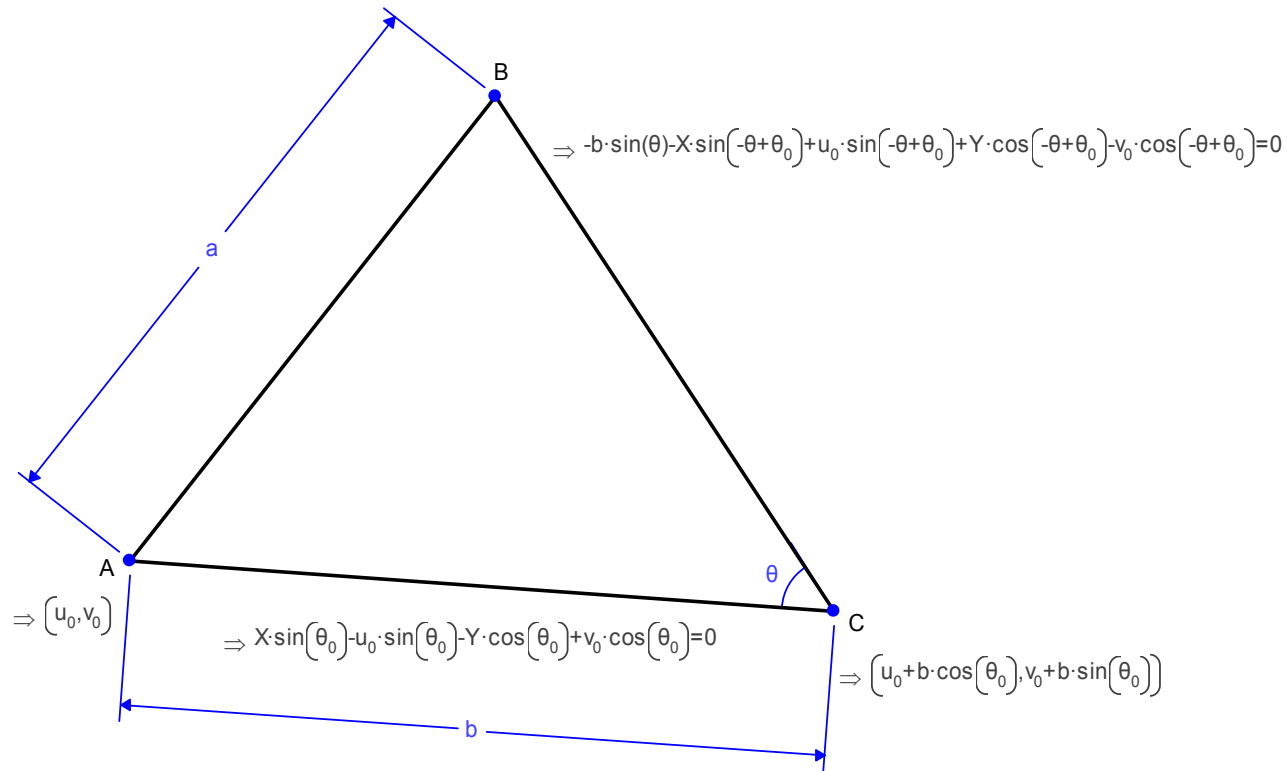


Architecture

- Archetypal Variable Value Manager
- ***Constraint graph algorithms***
- ***Algebra system***
- MathML

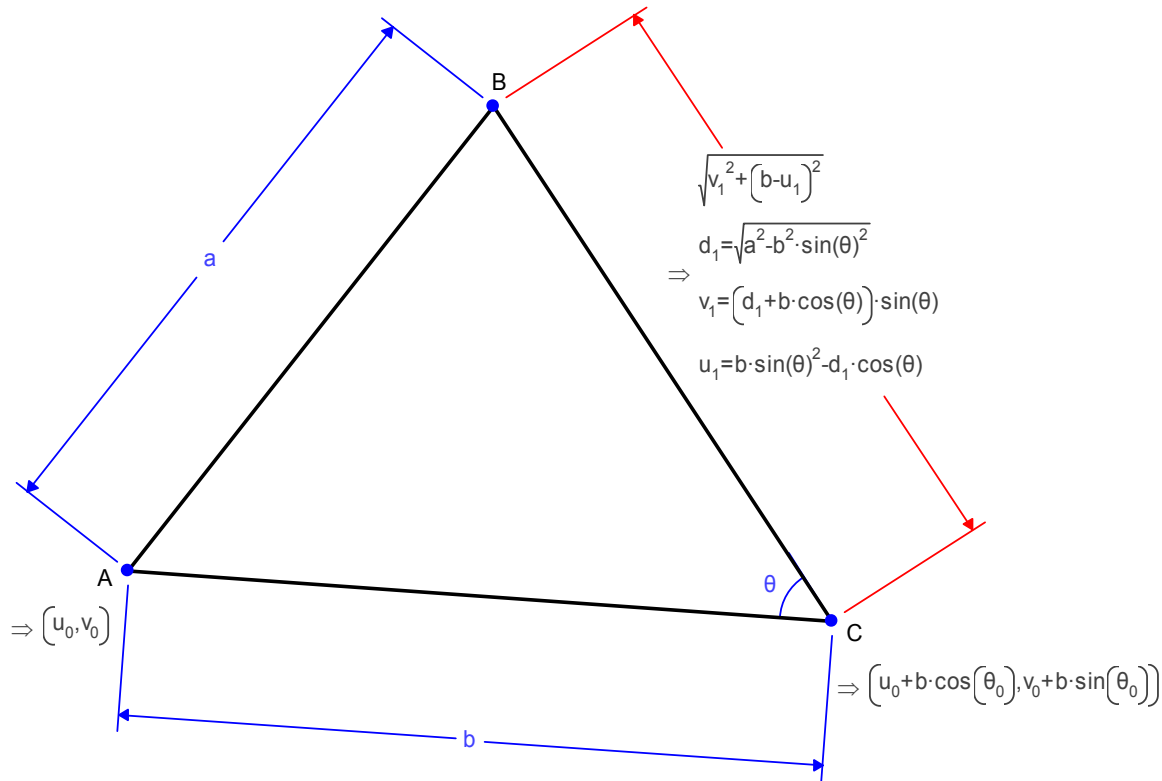
IV rich construction

$$\begin{aligned}
 & \left(u_0 - b \cdot \sin(\theta) \cdot \sin(-\theta + \theta_0) - d_1 \cdot \cos(-\theta + \theta_0), v_0 - d_1 \cdot \sin(-\theta + \theta_0) + b \cdot \sin(\theta) \cdot \cos(-\theta + \theta_0) \right) \\
 \Rightarrow & d_1 = \sqrt{a^2 - b^2 \cdot \sin^2(\theta)}
 \end{aligned}$$



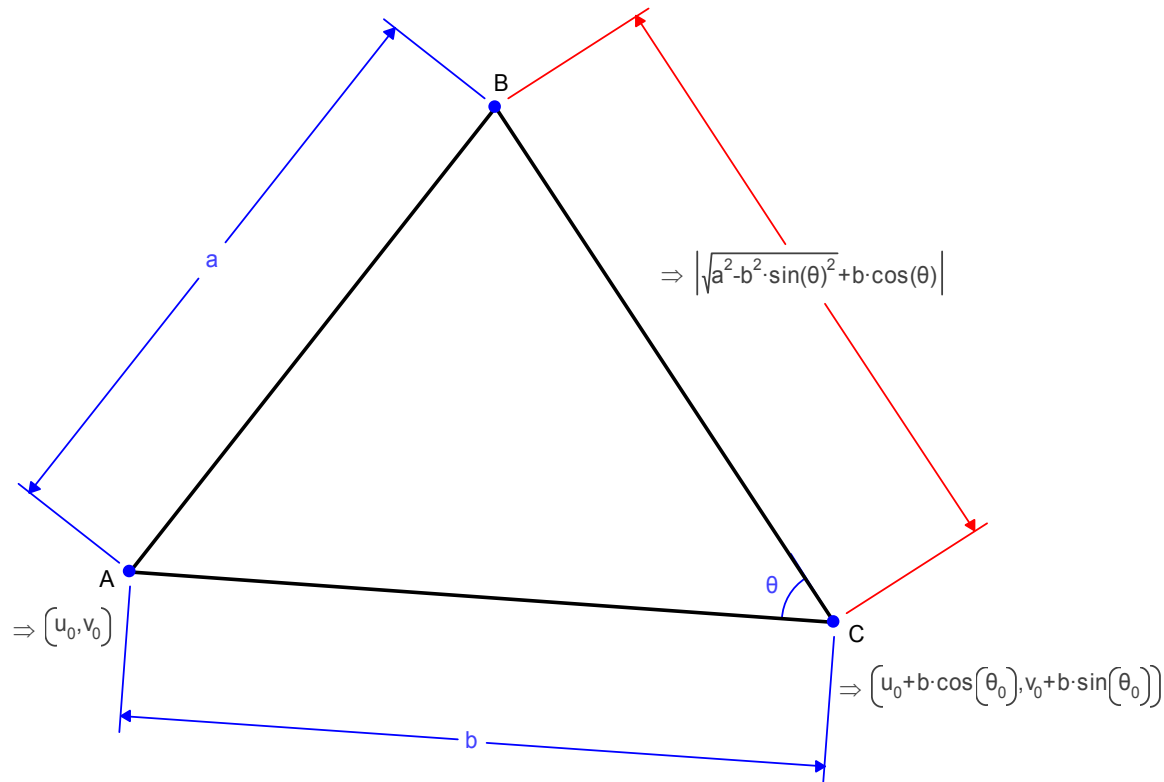
IV rich measurement

$$\begin{aligned}
 & \left(u_0 - b \cdot \sin(\theta) \cdot \sin(-\theta + \theta_0) - d_1 \cdot \cos(-\theta + \theta_0), v_0 - d_1 \cdot \sin(-\theta + \theta_0) + b \cdot \sin(\theta) \cdot \cos(-\theta + \theta_0) \right) \\
 \Rightarrow & d_1 = \sqrt{a^2 - b^2 \cdot \sin(\theta)^2}
 \end{aligned}$$



IV Removal & Simplification

$$\begin{aligned}
 & \left(u_0 - b \cdot \sin(\theta) \cdot \sin(-\theta + \theta_0) - d_1 \cdot \cos(-\theta + \theta_0), v_0 - d_1 \cdot \sin(-\theta + \theta_0) + b \cdot \sin(\theta) \cdot \cos(-\theta + \theta_0) \right) \\
 \Rightarrow & d_1 = \sqrt{a^2 - b^2 \cdot \sin^2(\theta)}
 \end{aligned}$$

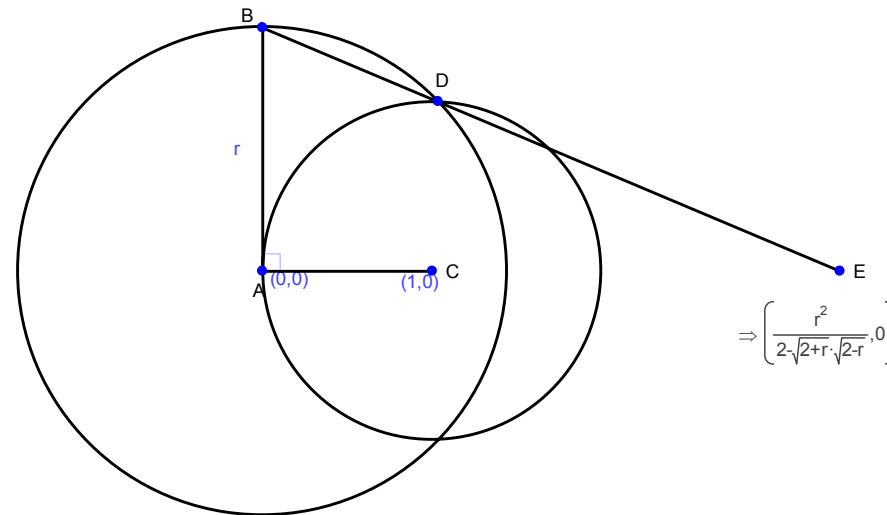




Architecture

- Archetypal Variable Value Manager
- Constraint graph algorithms
- Algebra system
- ***MathML***

MathML Export Import



$$\Rightarrow \left[\frac{r^2}{2\sqrt{2+r}\sqrt{2-r}}, 0 \right]$$

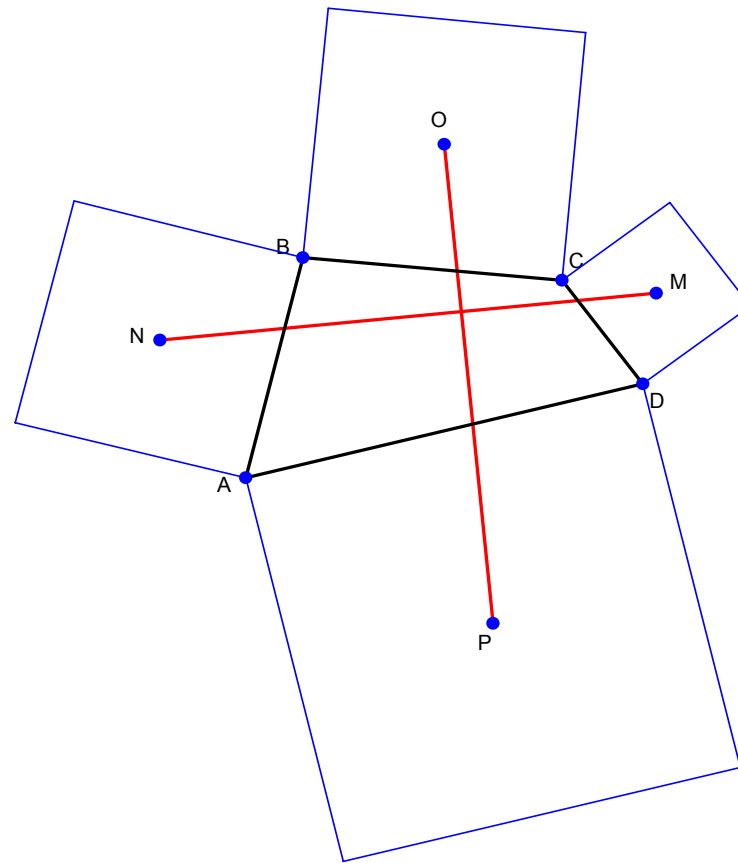
```
> limit(-r^2/(sqrt(-r+2)*sqrt(r+2)-2),r=0);
```



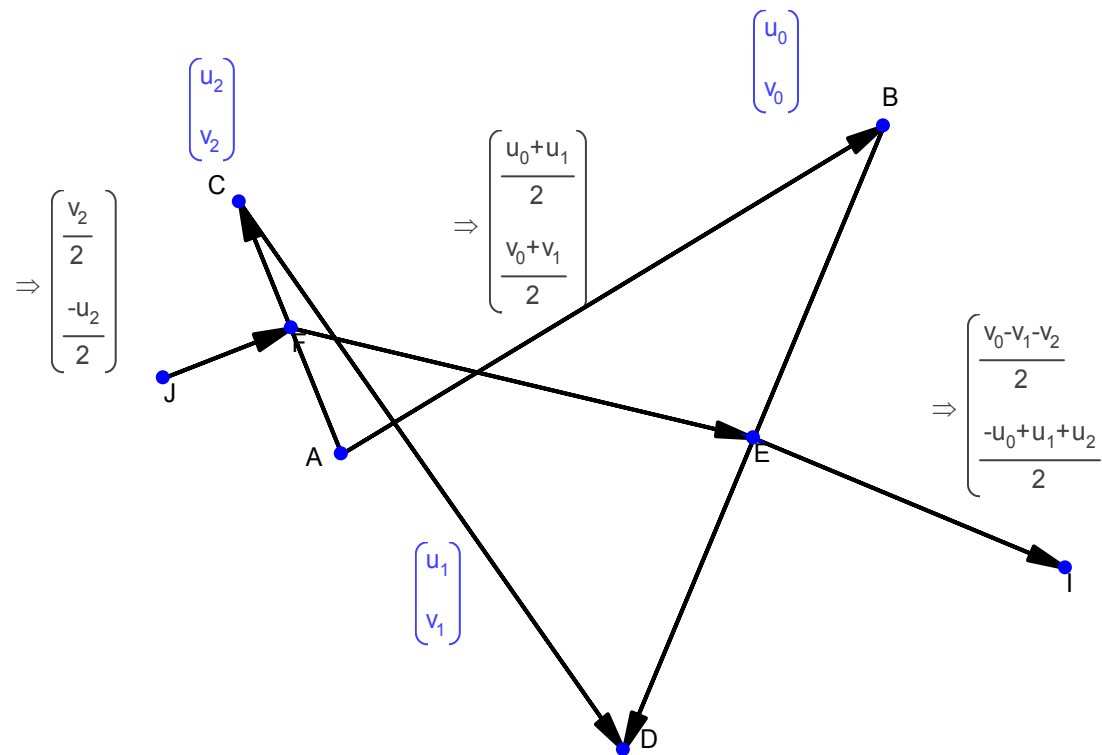
Outline

- Project History (1987 – 2006)
- Architecture of an Interactive Symbolic Geometry System
- ***Examples of Use***

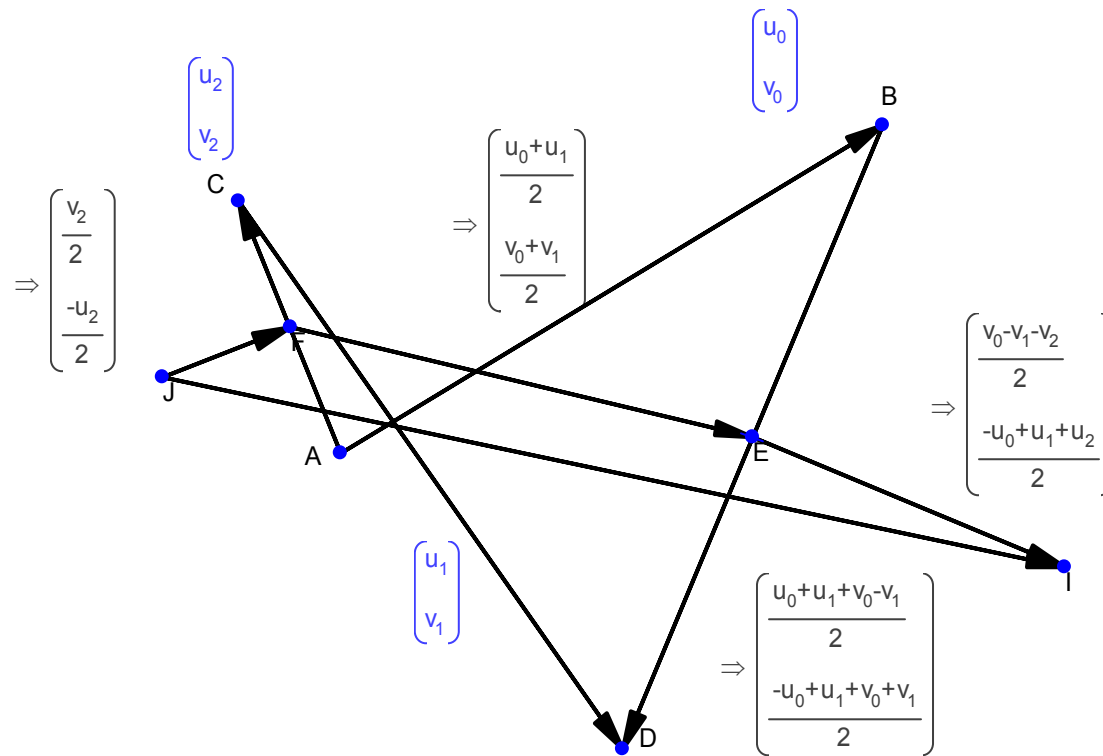
Example – von Abuel's Theorem



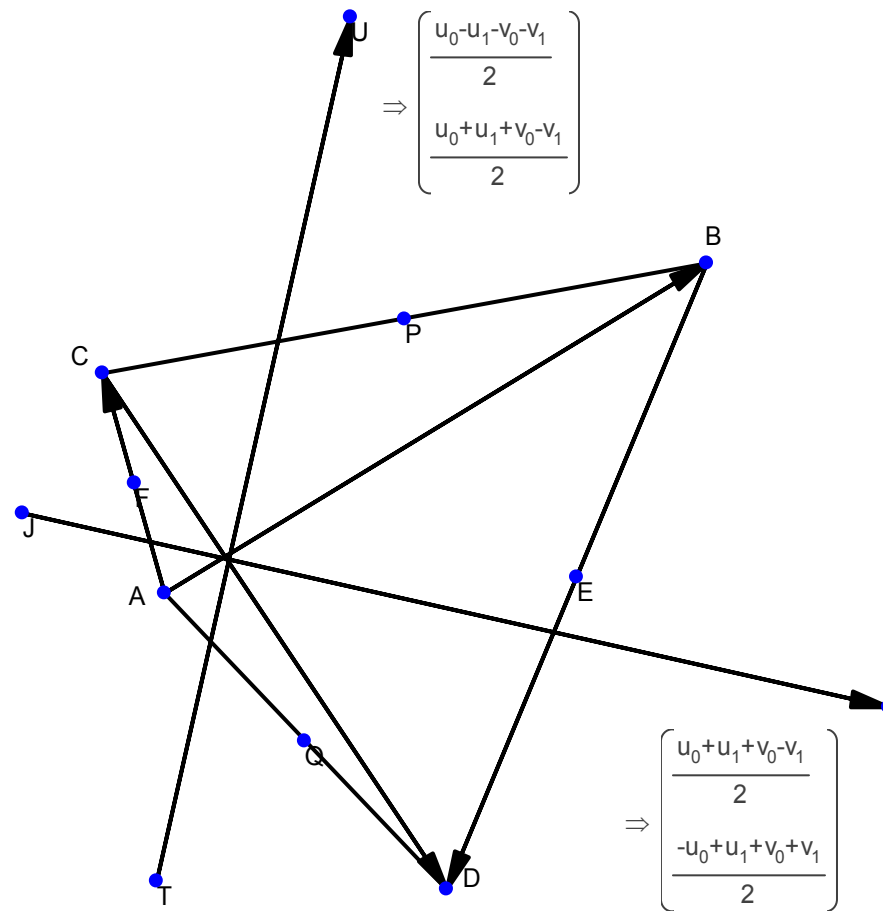
Von Abuel's Proof



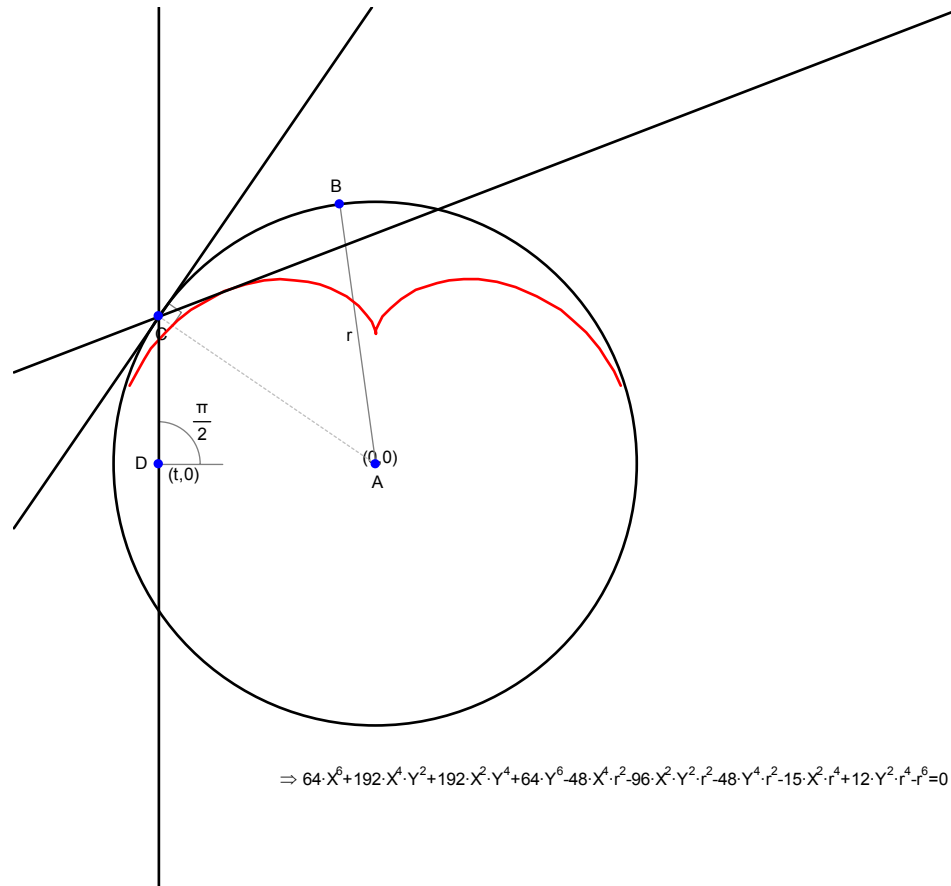
Von Abuel's Proof



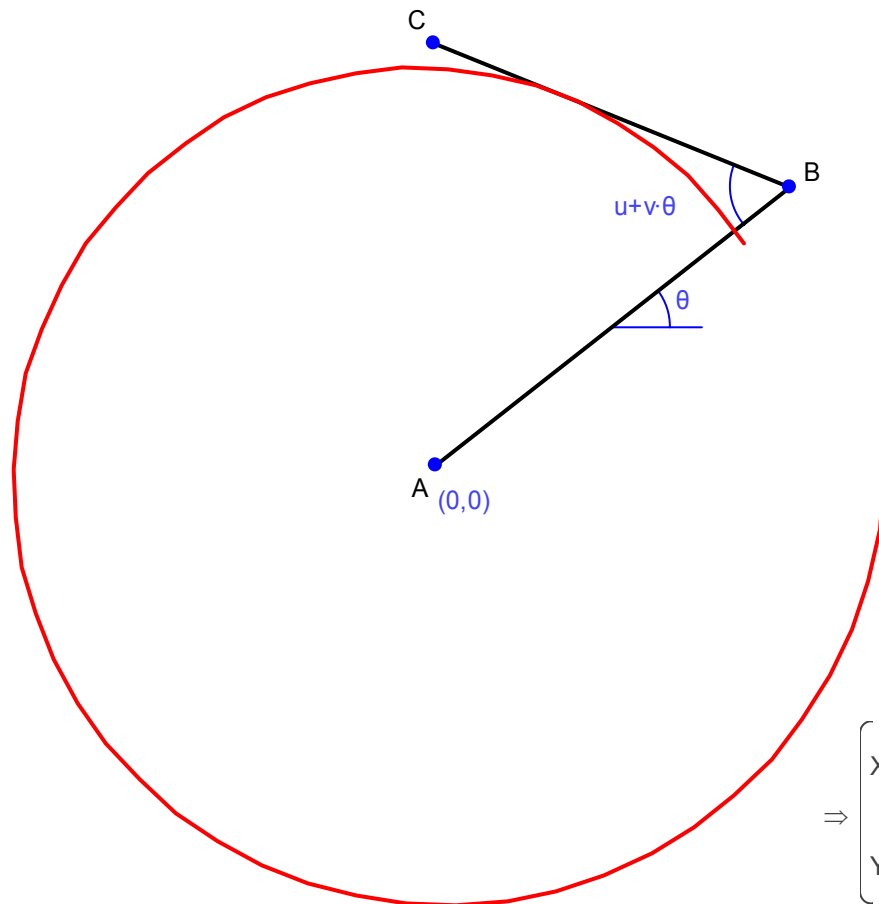
Von Abuel's Proof



Coffee cup caustic

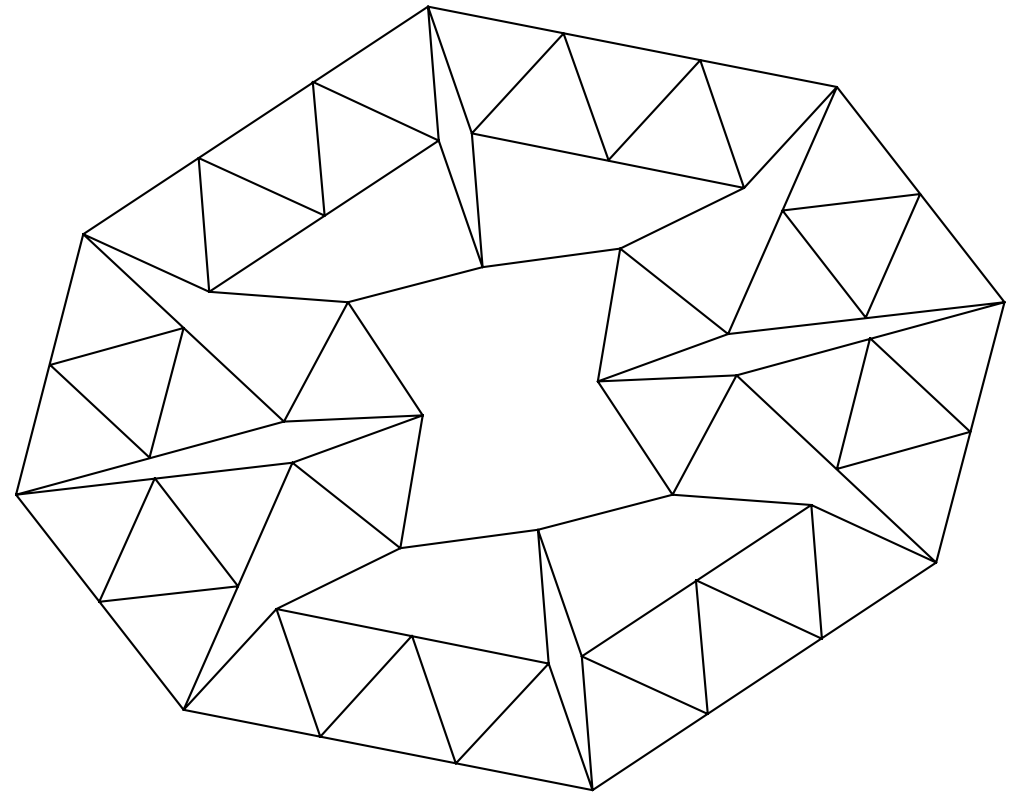
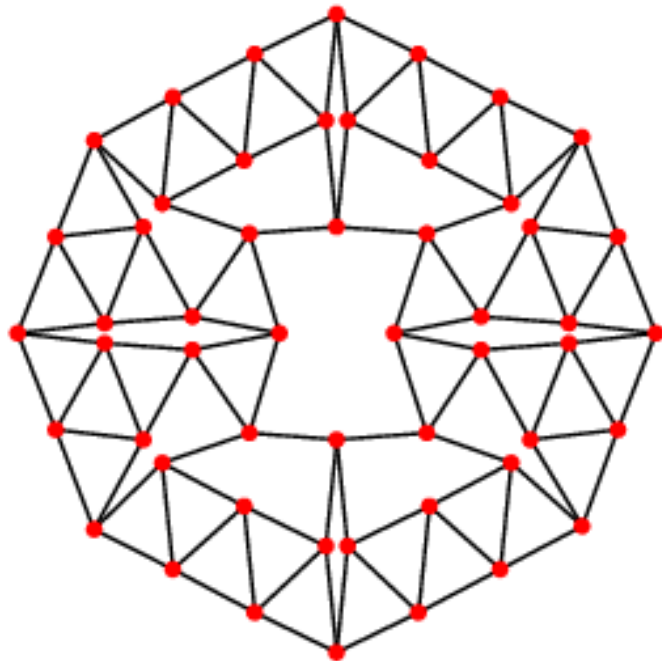


Linear rise flat face oscillating cam

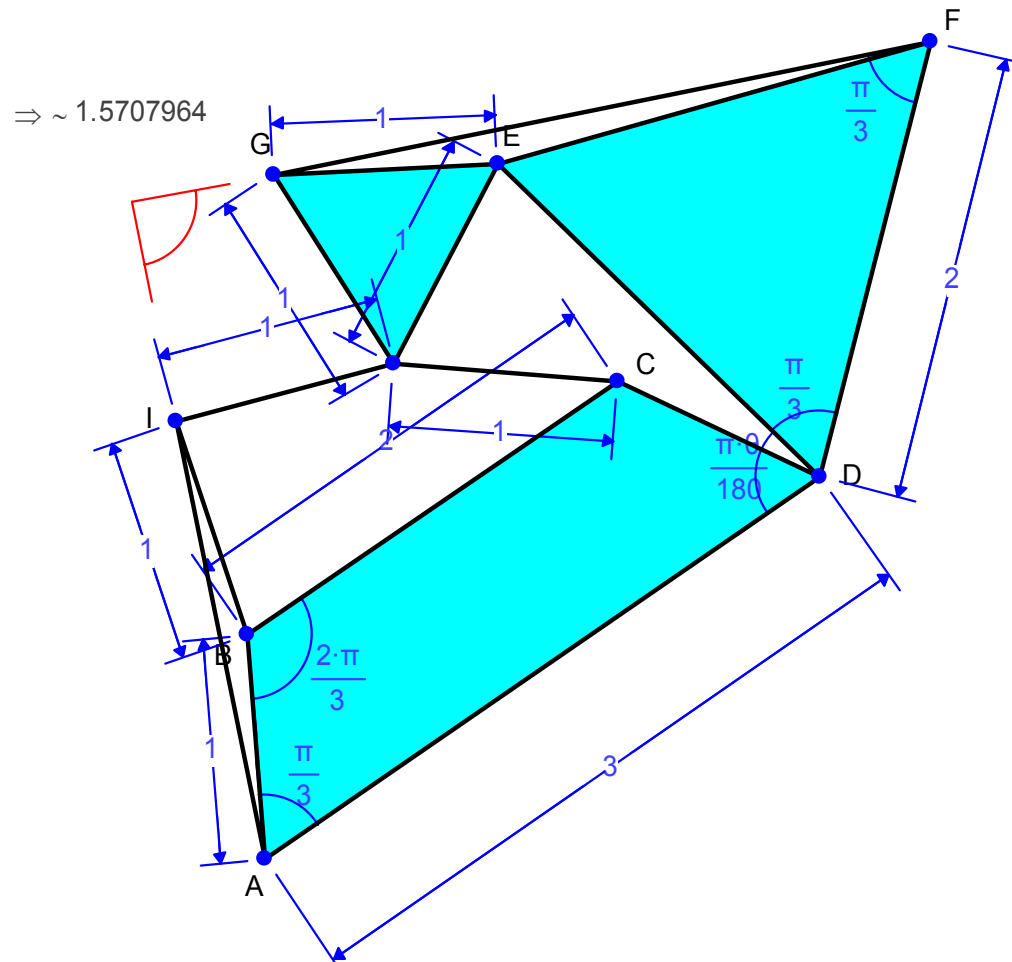


$$\Rightarrow \left[\begin{array}{l} X = \frac{d_0 \cdot (-\cos(\theta) + 2 \cdot v \cdot \cos(\theta) + \cos(2 \cdot u + \theta \cdot (-1 + 2 \cdot v)))}{2 \cdot (-1 + v)} \\ Y = \frac{d_0 \cdot (-\sin(\theta) + 2 \cdot v \cdot \sin(\theta) - \sin(2 \cdot u + \theta \cdot (-1 + 2 \cdot v)))}{2 \cdot (-1 + v)} \end{array} \right]$$

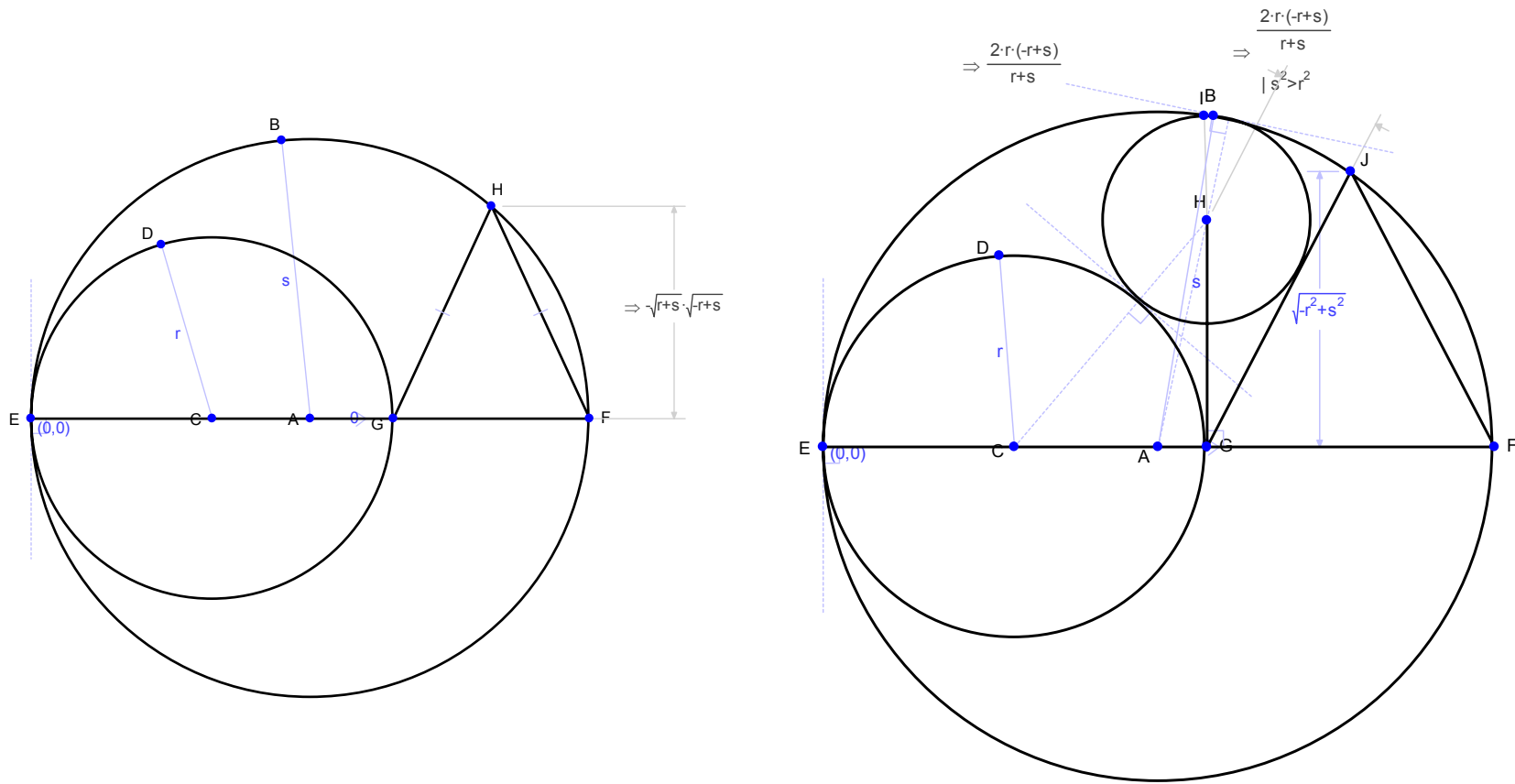
Harborth Graph



Harborth Derivation



Buehler's Circle Theorem





Geometry Expressions

www.geometryexpressions.com