On Using OpenMath for Representing Dynamic Geometry Constructions

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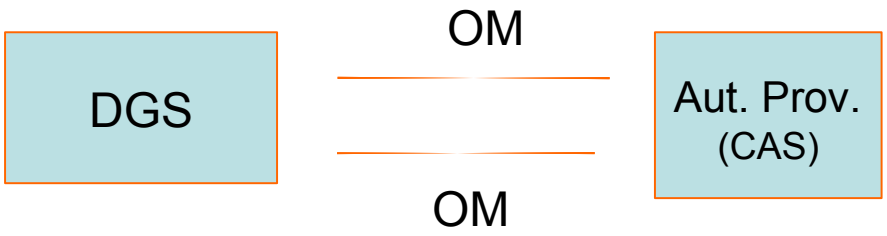
Overview

- **Goal of the talk:** To enhance reasoning habilities of (known) Dynamic Geometry Systems (DGS), connecting them with (known) external automated reasoning tools (Automated Provers), in order to prove geometry theorems



- We need a different OpenMath *translator* for each DGS, but the communication language is independent: OpenMath

Structure of the talk

- Dynamic Geometry Systems
- Automated Deduction
- Mathematics communication: OpenMath
- From Cabri/GSP to OpenMath
- GDI and OpenMath
- Webdiscovery and OpenMath

Dynamic Geometry

- Software that allows accurate constructions of geometric configurations on a computer screen
- Key point: As some parts of a construction are moved, the whole construction is automatically updated, preserving all relationships and constraints
- Example: Pappus theorem
- Dynamic Geometry Systems (DGS)
  - 80's: Cabri Géomètre (France), Geometer's Sketchpad (USA)
  - 90's: Cinderella (Germany), GEX (China), GEOTHER (France),…
  - 2002: KIG (GNU/Linux)

Automated Deduction with Dynamic Geometry

- Some Dynamic Geometry Systems use their own code to handle proof tasks
  - Numerical methods: **Cabri Géomètre**
  - Probabilistic methods: **Cinderella**
  - Symbolic methods: **GEX** (Geometry Expert)
    - Wu's, Groebner Basis, Vector, Full angle, Area methods

- Numerical approaches to *proving* theorems lead to mistakes
  - Cabri does not "see" changes in coordinates: .txt file, Cabri file
  - A more complicated example

Automated Deduction with Dynamic Geometry

- Other DGS use external CAS
  - **GEOTHER** (GEOmetry THeorem provER)
    - Implemented in Maple
    - Methods: Wu's, Groebner Basis, Wang's
  - **GDI** (Geometría Dinámica Inteligente)
    - Developed by Valcarce and Botana
    - Uses CoCoA and Mathematica

- To communicate DGS and CAS in a standard way, there is a need of a "Computer Math" communication language

Mathematics Communication (web)

- Standards in information communication
  - HTML
  - XML (recommendation in 1998 from the W3C)
- Standards in Math communication
  - MathML (basically XML oriented towards Math representation)
  - GOOL (meant to describe geometric configurations)
    - Developed by Wang and Liang (presented in ADG 2004)
    - Yet under development.
- Need for a semantics based language
  - Database storing and searching
  - Computations involving different applications

OpenMath

- Project started in 1993
- Extensible standard for representing the semantics of mathematical objects (example)
- Main elements: phrasebooks (“translators”)
- Example: Wiris Editor (presented at ICM2006)
- Example: Roozemond (TU Eindhoven)
  - OM to add proving capabilities to Cinderella using GAP (prototype)
- OM is not available in the new version of Cinderella, despite announcements

OpenMath to encode Geometry Constructions

- Everything that can be described with OM is based on the atomic elements contained in the Content Dictionaries (CD's)
- http://www.openmath.org/
- The list of CD´s is extensible and open
- Plangeo1, …, Plangeo6
  - Developed for Cinderella 1
  - Experimental = yet being tested
    - Example: not name in segment
- Example: Perpendicular Bisector in OpenMath

From Cabri/Geometer's Sketchpad to OpenMath

- Applets that produce the OpenMath description of ruler and compass constructions generated with
  - Cabri Geometry II plus
  - Geometer's Sketchpad
- Admit constructions with one of the five questions allowed by Cabri: *Is On? Parallel? Perpendicular? Collinear? Equidistant?*
  - With Cabri: directly
  - With Geometer's Sketchpad: added with the applet menu
- Applets available at
  - http://nash.sip.ucm.es/appletsOM/
- Example: Pappus.gsp, Pappus.txt

GDI and OpenMath

- GDI is a prototype of a Dynamic Geometry System (2003)
- It has been enriched so it can export the OpenMath representation of any construction allowed by the system
- Example: a triangle
- Possible to send questions to any OpenMath–understanding prover.

webDiscovery and OpenMath

- webDiscovery is a webMathematica based tool for remote proof and discovery tasks
- Originally designed to accept constructions from GDI, it has been modified to *understand* OpenMath

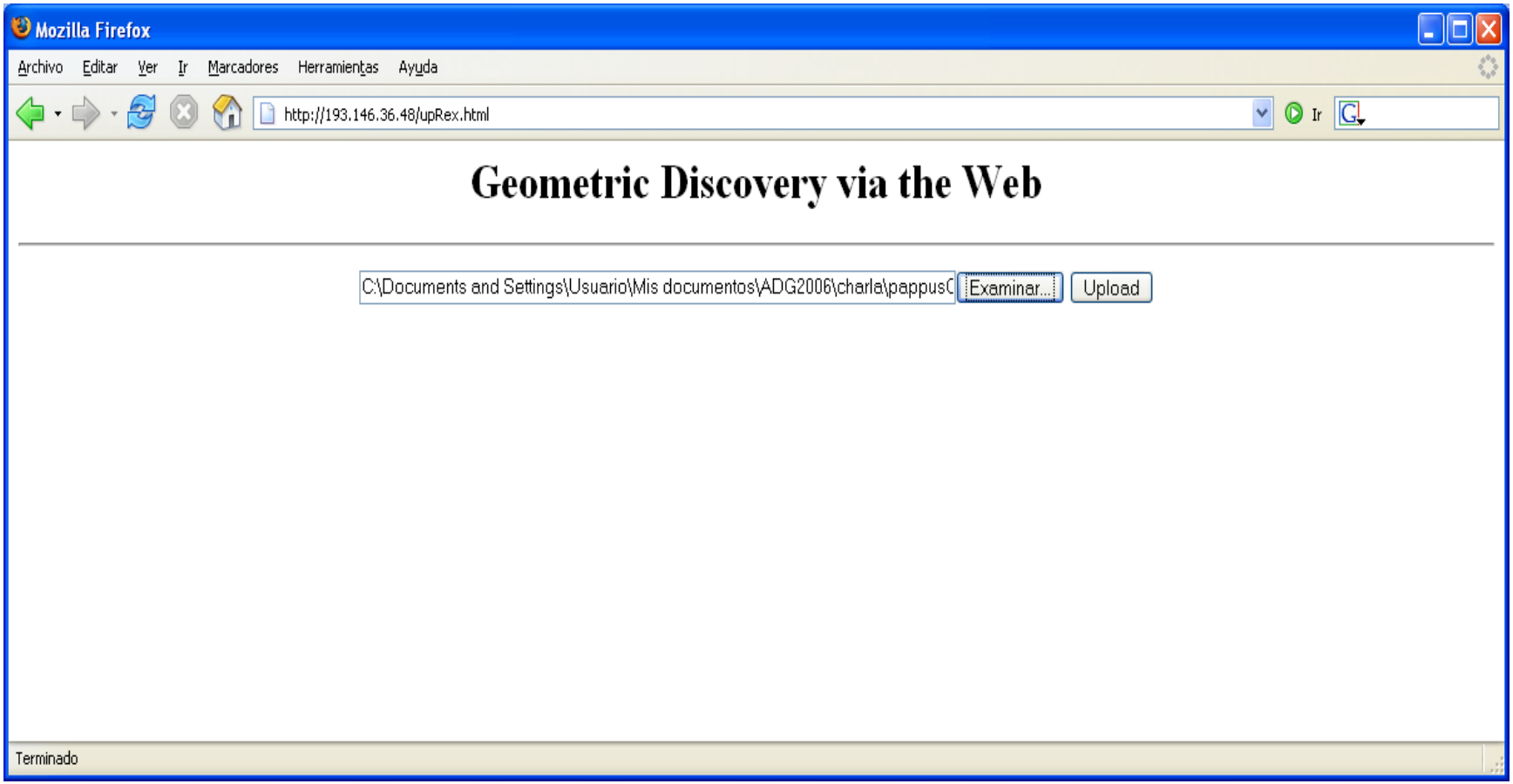
- Example:
  - Proof of Pappus theorem
    - PappusOM
    - Webdiscovery

Last remarks

- There is a clear need for a common “computer Math” language
- Example shows real potential of OpenMath to intercommunicate different applications

- Future work
  - Direct communication between DGS via OpenMath
  - Development of prover independent of commercial CAS

WebDiscovery input



Webdiscovery output